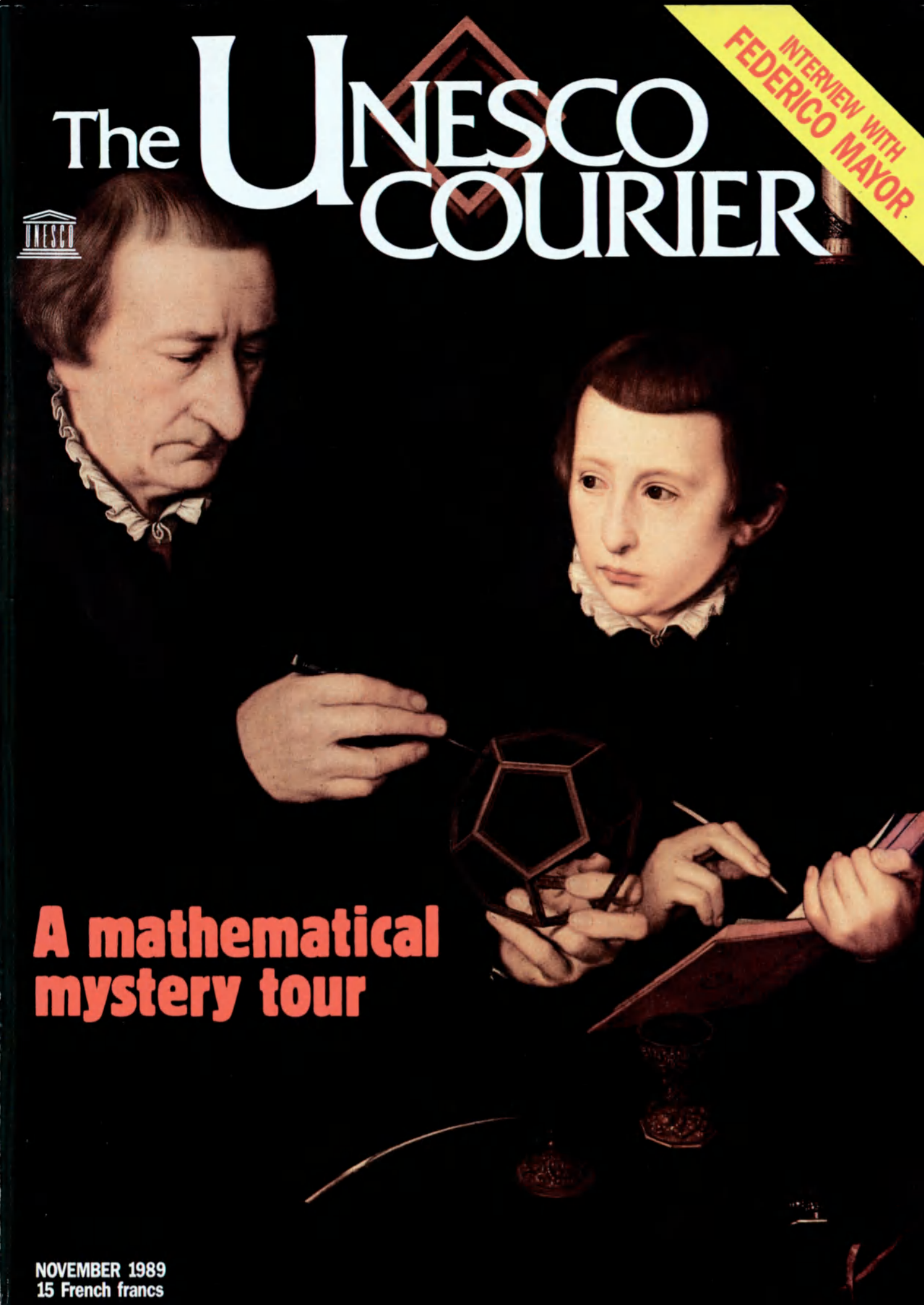
The UNESCO COURIER

INTERVIEW WITH FEDERICO MAYOR



A mathematical mystery tour

NOVEMBER 1989
15 French francs

encounters

We invite readers to send us photographs to be considered for publication in this feature. Your photo should show a painting, a sculpture, piece of architecture or any other subject which seems to be an example of cross-fertilization between cultures. Alternatively, you could send us pictures of two works from different cultural backgrounds in which you see some striking connection or resemblance. Please add a short caption to all photographs.

Buddha Shakyamuni
1987, painting on cotton executed with crushed natural pigments, by Nathalie Gyatso

The art of *thang-ka*, a form of Tibetan religious painting or drawing on woven material, is governed by strict rules. "But," writes the French-born artist Nathalie Gyatso, "there is room for creativity and invention in *thang-ka* art. Here, for example, I wanted to symbolize the calming effects of the Buddha's appearance by showing aggressive clouds retreating before the light, and vegetation overgrowing massive, threatening mountains. While following the same rules, a Tibetan painter would have made a different iconographical choice, perhaps associating peaceful motifs with a peaceful deity."

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SCIENCE AND SOCIETY
AN ACADEMY OF SCIENCES FOR THE THIRD WORLD
by Akhtar Mahmud Faruqui

Today there are no more unexplored continents, unknown seas or mysterious islands. But while we can overcome the physical barriers to exploration, the barriers of mutual ignorance between different peoples and cultures have in many cases still not been dismantled.

A modern Ulysses can voyage to the ends of the earth. But a different kind of Odyssey now beckons—an exploration of the world's many cultural landscapes, the ways of life of its different peoples and their outlook on the world in which they live.

It is such an Odyssey that the *Unesco Courier* now proposes to you, its readers. Each month contributors of different nationalities will provide from different cultural and professional standpoints an authoritative treatment of a theme of universal interest. The compass guiding this journey through the world's cultural landscapes will be respect for the dignity of man everywhere.

The Editors wish to thank Mr. Tony Lévy for his help in the preparation of this issue.

Cover:
The Geometry Lesson (1561), by the Flemish painter Nicolas de Neuchâtel.

Back cover:
Computer image generated using a formula from the new field of fractal geometry.

Federico Mayor

Federico Mayor was elected Director-General of Unesco for a 6-year term in 1987. He had previously served as rector of the University of Granada, president of the Spanish Society of Biochemistry, founder-director of the Madrid Centre for Molecular Biology, Spain's Minister of Education and Science, and as a member of the European Parliament in Strasbourg. On the occasion of the 25th session of Unesco's General Conference, he sets forth here his reflections on the role of the United Nations system in the world today.*

Although public opinion in most countries may be aware of the existence of the United Nations system, it is not always very sure of the system's use in practical terms. Do you really think that the UN system is indispensable?

— The system is even more necessary today than it was in the past. In 1945 it responded to a bitter, lucid analysis (lucidity and bitterness often go together) made in the aftermath of a disaster. Today it responds to an acute awareness of the increasingly global nature of problems which we can only face together.

The system has not solved all the problems with which it has been confronted, far from it. But it has shown itself to be increasingly irreplaceable as a meeting place, as a forum of conciliation and reconciliation, as a place where new approaches can be sought, and avenues of co-operation carefully explored. It is, fundamentally, the seat of a new identity, that of the "global village". A pluralistic and multiform village which regroups, and must protect, all cultural identities, the infinite diversity of peoples, and the irreplaceable originality of each community.

Contrary to what is often thought, the United Nations system was not constructed ex nihilo, arbitrarily established in 1945 on the decision of a handful of idealistic visionaries. It is the prolongation of a long history. It is the outcome of a historical process that began in the late nineteenth century whereby an increasing number of individuals, currents and nations have become aware of the need to create common structures of consultation and co-operation on an international scale.

This awareness was a response to new realities: people were starting to realize that the world was one: communications were intensifying; commercial, industrial and financial interests were no longer confined within national and even continental frontiers; information was beginning to cross the oceans on a regular basis. This emergence of a world system called for the organization of mutual consultation on a worldwide scale.

So the system was created in response to a need?

— The human species is the only one which is endowed with the distinctive faculty of creativity. There are always some far-seeing persons who take timely initiatives in response to new challenges.

The first institutional international structure—the League of Nations—was created after the First World War. This terrible convulsion revealed that the world was not only a market to be shared but a heritage to be protected. Some statesmen, but also philosophers, scientists and writers, thought then that it would be useful to have two distinct organisms, one to settle political differences (the League of Nations) and the other to promote intellectual and cultural co-operation (the International Institute of Intellectual Co-operation).

Of course, when I speak of heritage I am not only

* Author of many scientific publications, several books of poems, and an essay entitled *Mañana siempre es tarde* (Espasa Calpe publishers, Madrid, 1987).

thinking of our physical heritage—the natural, environmental, artistic and architectural heritage—but also of our common spiritual and intellectual heritage—the fruits of knowledge, human rights, the universal values and principles.

But the League of Nations was not strong enough to resist the hurricane of fascism in the name of all this. The Second World War was a much more serious warning for humanity. Not only were the death and destruction it caused out of all proportion to those of the earlier war; not only did the scale of the conflict gradually extend to five continents; in addition, the explosion of two atomic bombs showed to everyone that for the first time in history mankind had endowed itself with the means of self-annihilation. A third world war could mean the extinction of the human race.

Until then, all wars—all conflicts—had had their winners and losers. Now all sides faced the threat of losing. War became a nonsensical proposition. Almost imperceptibly the civilization of war gave way to the civilization of peace, not through an access of virtue but through fear of the power now commanded by the technology of destruction. The culture of peace was to be born, not of the wisdom of men but of their infinite distress. It was at this turning point—to my mind perhaps the most important in human history—that the United Nations system was established.

Much more ambitious than the League of Nations, the UN set itself the goal of extending the field of co-operation to all nations and to many fields of intellectual, social, economic and humanitarian activity. The United Nations Organization is the political organ of this system, which is completed by a range of international organizations, agencies, programmes, funds and commissions: Unesco for education, science and culture, the WHO for health, Unicef for children, the ILO for problems related to work, the High Commissioner for aid to refugees, the FAO for food and agriculture, etc.

Of course, the successes great and small, the spectacular and less spectacular achievements, the inadequacies or the failures, of the organizations of the United Nations system are closely dependent on the state of mind of the community of nations and especially on the aptitude of the most influential states to agree to ensure that a desire for conciliation prevails. If these favourable factors exist, the United Nations family can fully exploit the dynamics of multilateral co-operation and promote the elaboration in common of solutions which cannot be found individually.

This, I think, is the real purpose of the work of the United Nations. The UN is there to remind us that we belong to a single species and inhabit a single planet. What would become of us if we were ever to forget this?

You speak of the organizations and agencies belonging to the United Nations system. Some people may accept the need for institutions to promote international consultation with a view to co-operation and regulation in politics, health, agriculture or financial affairs, but what about culture? Is there not a contradiction between the idea of organization—with its rules, programmes and choices—and the idea of culture, which is by definition synonymous with creation, that is, with freedom and spontaneity?

— Clearly Unesco's mission is not to intervene in the processes of creation but to encourage the conditions in which cultural activities can develop and bear fruit. But first of all we must distinguish between some different meanings of the word culture which often tend to be confused.

At first culture was set in opposition to nature, to differentiate man the creator from animals which can only obey natural laws. The word has also been used to distinguish between the education of the mind and the production of goods by manual labour. Different definitions of culture have been formulated, some extensive, some less so. For some, culture only includes the sublime masterpieces of thought and creation; for others, culture englobes everything—from the most sophisticated products to beliefs, customs, ways of living and working—which differentiates one people from another. This second meaning was adopted by the international community at the intergovernmental Conference on Cultural Policies which was held in Venice in 1970. In 1982 a second Conference, "Mondiacult", which was held in Mexico City, ratified this "active" approach.

This definition has been criticized by some intellectuals on the grounds that it may tend to privilege the most trivial rather than the highest expressions of the mind, and to stress the values specific to each people—those which separate them from others—instead of the universal values of Truth, Good, Beauty—which bring men closer together.

— I think this criticism is aimed at the wrong target. It is based on a misconception of the frame of reference of our definition, which is of a practical, operational order. In the United Nations system, in which different philosophies coexist, the bases of agreement are bases of action, of action which is of course guided by the major principles on which the entire system rests.

Definitions are points of departure. And in cultural co-operation, the only possible point of departure is recognition of diversity and respect for this diversity—which is the basic form of respect for the dignity of individuals in different cultures. More specifically I will say that each person is biologically and socio-culturally unique and that recognition of this uniqueness is the sine qua non of mutual understanding. But this certainly does not mean that everything in all cultures is of equal worth, and that it is impossible to transcend cultural specificities. There is no naivety in Unesco's approach: cultures are not untouchable entities; they are the product of a history. Each one has its highlights and its dark corners, its grandeur, its impulses towards the sublime and its dreams of fraternity, but also its less positive inclinations. Our role is to appeal to what is best and most fully human in all of them, to what, naturally, produces a spirit of peace.

Given these considerations, how would you define the purpose of Unesco's work?

— It is to cut across cultural pluralism and give weight to whatever brings men closer together, whatever is universal in each culture. To do this, it is necessary to begin at the beginning, with the basic idea that each culture, which is often in its turn a melting pot of other cultures, has its own genius, its own trademark in the history of human civilization, the identity mark whereby it recognizes itself at each moment of its trajectory, and whereby it can also be recognized by the rest of the world.

I am a Catalan; I profoundly love my country and the language in which I have always addressed my parents. This culture, the fruit of several civilizations, defines our specific profile. I know that only if I can develop it without hindrance, can I work for the unity of Spain, for the strengthening of all the other national cultures. In the past, ignorance of this fundamental truth has often been at the roots of contempt for others. There are grounds for thinking that in the future mutual consultation will become easier to the extent that peoples will have learned to know themselves; to understand their motivations, their ways of being and their respective scales of values; and to discover that over and above these specific expressions they share great common aspirations and ideals which often coincide on essential matters.

This is why Unesco's primary mission in the field of culture is naturally, as expressed in its Constitution, "to develop and to increase the means of communication between ... peoples ... for the purposes of mutual understanding and a truer and more perfect knowledge of each other's lives".

Unesco seeks to promote contact and meetings between artists, craftsmen, intellectuals, painters, educators, architects, writers and poets from all regions and all countries, and to publicize the discussions and conclusions that emerge from these encounters. These people constitute the principal wealth of humanity. They can not only enlighten us about the meaning of the past but also and above all about the possible shape of the future. It is in collaboration with

non-governmental organizations, associations of professionals, artists and creators that Unesco draws up and executes its cultural programmes.

I never tire of saying that the most perfect monument is the human being; that the most precious—and the most threatened—cultural works are the minority languages, the oral traditions, the songs, dances and customs of the many countries which do not yet participate in the great cultural concert of the world. I strongly insist on this. Just as the freedom of artists must be protected, so the survival of the arts, folklore and popular cultures must be ensured. These two imperatives go together.

Major Unesco activities such as the rescue of Abu Simbel and Borobudur are well known. Today there is much talk of an agreement between the different Cambodian parties concerning Unesco's role in the conservation of Angkor Vat.
— Unesco is ready to act as soon as circumstances permit. But its work in the field of culture goes much further than the rescue of monuments and sites. Unesco administers three international juridical instruments of major importance: the Convention for the Protection of Cultural Property in the Event of Armed Conflict; the Convention on the Means of Prohibiting and Preventing the Illicit Import, Export and Transfer of Ownership of Cultural Property; and the Convention concerning the Protection of the World Cultural and Natural Heritage.

Unesco is also concerned with the translation and publication of literary masterpieces from many countries, with listing, recording and diffusing a unique collection of pieces of music from all over the world.... In many countries today, books, records and radio and television broadcasts give increasing exposure to works from other countries, to the realities and problems of the most diverse regions. Slowly but surely the universal is entering the daily life of everyone. This is one of the important and appreciable aspects of Unesco's work.

What of cultural creation in the strict sense?
— It is and can only be the province of individuals who are absolutely free to follow their own inspiration, conscience and talent. Nothing should thwart or censor this freedom. On the contrary, everything possible should be done to protect it and allow it to flourish. I would say that the place for Unesco's action is before and after this freedom. On the one hand, it is concerned with denouncing possible violations. On the other, with encouraging the optimal conditions for its creative exercise.

It is clear that the free circulation of ideas and works from all cultures is one of the conditions most favourable to the blossoming of creativity, to the stimulation of learning, to the broadening of horizons and the mutual enrichment of the sources of inspiration available to every artist. At the same time it fosters a more profound understanding of the universal significance which artists can find in their own cultures.

The relationship between deeper mutual knowledge, increasing cultural exchanges and the strengthening of peace is not obvious to everyone. There is a striking number of conflicts between neighbouring peoples which know and communicate with each other. And the two world wars broke out in Europe, between peoples who shared essential cultural values and benefited from advanced means of communication.
— It is often neighbouring peoples, neighbouring cities, which find mutual understanding most difficult—because they confront each other face to face and each is tempted to become immersed in its own traditions and despise those of others. Mutual understanding is not automatically produced by the political or commercial contacts established between peoples. These contacts may just as easily lead to conflicts as to agreements, depending on whether the reflexes of fear, suspicion, and mutual contempt are fostered, or the aptitude, latent in all peoples, to appreciate truth and beauty wherever they may be found, to respect the diversity of ideas and styles, to prefer tolerance and seek conviviality.

Take the nazi ideology, for example. How was it engrafted onto German culture? By perverting the noblest aspects of that culture, by exalting only the fibres of self-love. The nazi leaders ignored the humanism of Kant, the universalism of Goethe, the generosity of Beethoven. They systematically proclaimed racial superiority and the morbid cult of violence, striving to accustom their compatriots to the idea that all German history led naturally in that direction. Similarly they also had to falsify German history by reducing it to racial confrontation; by keeping alive memories of past wars and emphasizing everything that denigrated others, to whom they imputed the responsibility for Germany's misfortunes; thereby justifying future revenge. Today all that seems mad, unreal.

In this respect, it is interesting to recall an experiment in which Unesco was directly involved. Just after the war, one of the tasks our Organization embarked on was the revision of history books through the identification of factual errors and biased judgements. Historians from both sides in the recent conflict took part in the project. They got down to work with plenty of good will. But the experiment was a failure. They differed not only on the interpretation of the facts, but sometimes on the very existence of certain facts.

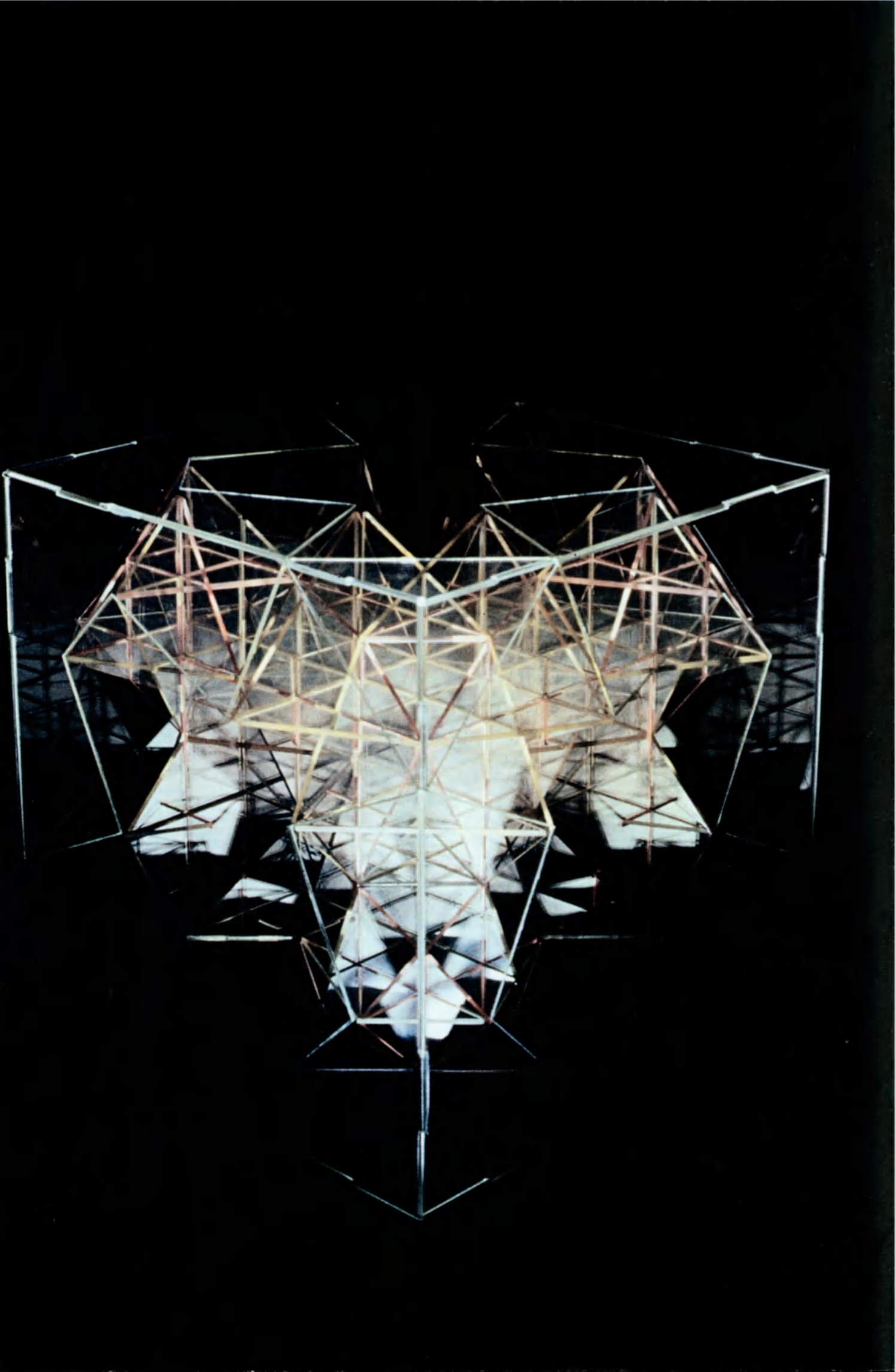
What conclusions do you draw from this? Had the war left so deep an imprint on people's minds that it was unrealistic in its immediate aftermath to hope to achieve an objective view of things?

—I think it was much too soon. But the evil ran far deeper. As I suggested, the war had ploughed land which had already been prepared. It had opened far older wounds, dug deeper into furrows traced by earlier wars, by manipulating culture and history. The same thing has happened since in many other conflicts, even if their protagonists have not gone so far as to advocate a final solution.

War has deep roots in the past of many peoples. To extirpate these roots a constant and courageous effort in the service of truth is needed. Political leaders are naturally called on to play a decisive role in this process. But they are not alone. Philosophers, artists, film makers and journalists all contribute, whether they know it or not, to the extent that they arouse interest, respect and admiration for the culture of others as well as for their own.

Here Unesco can take the lead as an instrument of mobilization and movement on the world level. Scientists, and teachers in schools and universities, must also take part in this effort. The unfettered circulation of scientific and technological information throughout the world is an irreplaceable vehicle of intellectual complicity and practical mutual assistance between researchers of all regions. The exchange of experience between educators from all over the world is also gradually spreading the conviction that each one of us is the repository of a precious portion of truth.... And that no one, absolutely no one, possesses the whole truth. It is on this difficult borderline between doubts and certainties, between self-respect and respect for others, that we must stand if we are to serve freedom and promote a permanent creative tension.

On the eve of this new phase in world history which is opening beneath the emblem of a culture of peace, education and access to knowledge must become the common destiny of all without exception—no longer the privilege of certain persons in certain countries. This is our dream. The dream of a new page to be written in a new language formed day after day by free citizens expressing their thoughts and creativity without constraint. A language that reflects an authentically peaceful state of mind; a language shorn of all exclusivity and segregation. A language which may be at last, uniquely, a vehicle for culture. ■



EDITORIAL

MATHEMATICS is one of the most abstract forms of intellectual activity. And yet we use mathematical skills not only when engaging in speculative thought but when trying to solve many practical problems of our daily lives. The development of these skills in early times was closely connected with that of writing. In a word, mathematics forms part of our cultural background and history.

Many of us remember the concentration and application that were necessary when we made our first steps in the world of maths. The same tenacious application, allied to a passion for the subject, is characteristic of advanced students and professional mathematicians.

This issue offers readers—even those for whom the laws of arithmetic have always been a closed book—a guided tour of some of the highways and byways of mathematics at different times and places in world history.

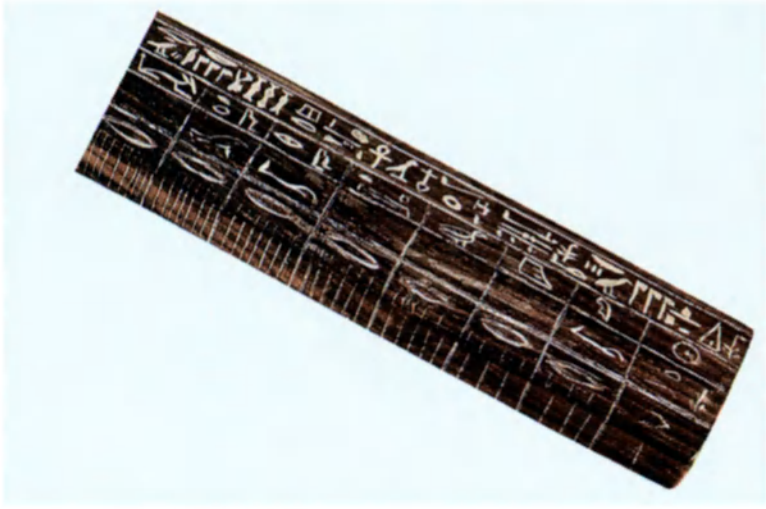
The assignment was a challenging one for our specialist contributors who were asked to give a succinct treatment of their subject in straightforward, non-technical terms. Wherever possible they have tried to put the history of mathematics in its social, cultural and even linguistic context.

Is there a common denominator, a single thread of reasoning which runs through all mathematical operations? There are many indications in these pages that this has often been the case. As the Greek philosopher Proclus remarked fifteen hundred years ago, mathematical reasoning is a demonstration of the unity that lies within multiplicity, the indivisible within the divided, and the infinite within the finite.

Union of the 3s in the numerical cube, by the Mexican artist Juan-Luis Díaz.

Prime numbers

BY JAMES RITTER



MATHEMATICS and writing have a close, symbiotic relationship. Indeed, recent archaeological discoveries have shown that it was the need to measure, divide and distribute the material wealth of societies that gave birth to the first writing systems.

For a society to develop a mathematics that goes beyond simple counting, a material support of some kind is essential. Without writing, the limitations of human memory are such that only a certain degree of numerical sophistication can be achieved.

By allowing us to follow the development of two writing systems, one in southern Mesopotamia towards the middle of the fourth millennium BC, and the other in the area around Susa in Iran, slightly later, archaeological discoveries in the last few decades have shown that the inverse is equally true. For a society to develop writing, material needs and, in particular, the need for record-keeping, are central.

In these societies the material support was clay, which is virtually indestructible, and the first documents are accounts. Cuneiform (wedge-shaped) Mesopotamian writing in particular was to know great success over the succeeding 3,000 years. Used to write not only the original Sumerian and Akkadian, but also Hittite, Elamite, Hurrian and many other languages of the ancient Near East, it died out only at the beginning of our era.

Meanwhile, an independent civilization developed rapidly in Egypt towards the end of the fourth millennium. Here the situation concerning writing is less clear. First of all, the material support, for other than monumental inscriptions, was principally papyrus, the reed-like plant growing along the Nile and in the Delta, and, to a lesser extent, other perishable materials. Egypt has thus yielded fewer documents than Mesopotamia by a factor of thousands.

Number systems

The third millennium marks, for both the Mesopotamian and the Egyptian civilizations, the gradual emergence of an abstract concept of number. Originally each number is attached to a given system of units. The "four" of "four sheep" and "four measures of grain" are not, for example, written with the same symbol.

Above, part of a rod used in ancient Egypt to measure the cubit, a unit of length equivalent to 525 mm.

Right, air view of the site of Ur (Iraq), dominated by the ziggurat (top).

Equally, the different systems of units are not connected among themselves. Measures of area, for instance, have no simple relationship to measures of length since the connection between the two—that area could be calculated from the product of a length and a width—was not yet operational.

But the very practice of writing things down, yielding permanent records of measures, opens up the possibility of observing regularities and patterns. This possibility was taken advantage of in these two societies over a period of about 1,000 years so that by the end of the third millennium Egyptian and Sumerian scribes had learned how to calculate areas and volumes from lengths, how to divide rations among workers, how to calculate the time necessary for a given job of work from volumes, numbers of men, and work rates. Next our evidence shows how a new level of abstraction was reached in which the concept of number became more and more detached from its metrological context.

By the beginning of the second millennium both civilizations had succeeded in developing equally abstract systems of numeration, though they had chosen different paths to represent their numbers. The Egyptians, like the vast majority of modern societies, had a written number system based on ten; that is, one counts up to nine of each unit before moving up to the next higher unit—after nine "ones" comes a "ten", after nine "tens" comes a "hundred", etc. But, unlike modern systems, their writing of numbers is "additive", that is there are separate signs for units, tens, and hundreds, and these are repeated as necessary.

Below, two scribes are shown recording an Assyrian king's spoils of war in this drawing of a lost neo-Assyrian wall painting.

The Mesopotamians used a base of sixty for their mathematical calculations, and they developed the first known system of position. The signs for numbers repeat after fifty-nine, the actual value being determined by the position of the digit in the number as a whole.

A scribe's training

Learning how to manipulate such number systems requires a specialized training and the creation of schools can be traced back to the same time as the invention of writing. Furthermore we know that the learning of arithmetic began early in the schoolchild's career, along with reading and writing, and that mathematics, then as now, was considered one of the "hardest" subjects.

Around 2000 BC, Shulgi, one of the kings of the Ur III empire in Mesopotamia, was the subject of a literary hymn which became a model text used as a school exercise during the first half of the second millennium. In this document he

Relief, above, from the tomb of Akhethotep (5th dynasty of the Old Kingdom, 2450-2290 BC) shows Egyptian scribes drawing up the accounts of a funeral precinct.

boasts of his academic achievements and proudly states: "I am perfectly able to subtract and add, [clever in] counting and accounting."

Over a thousand years later the Assyrian king Assurbanipal was to repeat much the same thing in one of his hymns: "I can find the difficult reciprocals and products which are not in the tables."

What would be the history of a young scribe destined to become a "mathematician" in ancient Egypt or Mesopotamia? He (and it is almost certain that it would be a boy; girls, while not forbidden a scribal formation, are almost entirely absent from the records) would of course first go to school. The sons of the rich and powerful rubbed shoulders with less fortunate young people, who found in education one of the rare means of social ascension.

What would the scribe learn during his schooling, which lasted at least ten years? We possess, from both civilizations, examples of school exercises including mathematical texts, and can catch a glimpse of student life through the so-called "scribal disputes". An example from Mesopotamia has one scribe boasting to another of his achievements:

"I want to write tablets:
tables [of measures] from 1 *gur* of barley to 600 *gur;*
tables [of weights] from 1 shekel to 20 minas of silver,
with the contracts of marriage that may be brought me,
the commercial contracts...
the sale of houses, fields, slaves,
pledges in silver, contracts for the rental of fields,
contracts for raising palm trees...
even the tablets of adoption; I know how to write all that."

In this typical text from Egypt one scribe taunts another as follows:

"You come here and fill me up with your office. I will show up your boastful behaviour when a mission is given to you. I will show up your arrogance when you say: 'I am the scribe, commander of the work gang.' ...

"A ramp, 730 cubits [long] and 55 cubits wide, must be built, with 120 compartments filled with reeds and beams; with a height of 60 cubits at its peak, 30 cubits in the middle; with a slope of 15 cubits; with a base of 5 cubits. The

Early Egyptian and Babylonian numerals

These two groups of symbols show how scribes of ancient Egypt and Babylon would have written 1989 using their respective number systems.

Egypt

1000 900 80 9

The Egyptian numeral system, here shown in hieroglyphic writing, was based on 10. We see *one* sign for thousands, *nine* signs for hundreds, *eight* signs for tens, and *nine* signs for ones.

Babylon

33 9

The Babylonian numeral system was based on 60. Using base 60, one thousand nine hundred and eighty-nine would be written as *thirty-three* sixties and *nine* ones. The position of the nine to the right indicates that it constitutes the ones, while the 33, one place over to the left, represents the sixties.

quantity of bricks is demanded of the troop commander.

"The scribes are all assembled but no one knows how to do it. They put their faith in you and say: 'You are a clever scribe, my friend! Decide quickly for us, for your name is famous.' ... Let it not be said: 'There is something he does not know.' Give us its quantity in bricks. Behold, its measurements are before you; each of its compartments is 30 cubits [long] and 7 cubits [wide.]"

But these are literary and not real mathematical texts. In fact, we possess some mathematical school texts from each civilization, almost all of them from two distinct periods: the first half of the second millennium and the period of Greek and Roman domination at the end of the first millennium. They are of two kinds, table texts and problem texts.

A typical example of a table text is a square root table from early second millennium Babylonia. The systematic and organized writing of a table shows the high degree of abstraction already attained. What does the scribe do if he needs a square root? If it is not on the table, it can easily be calculated from the surrounding values that do appear. This procedure was standard in almost all cultures including Western ones until a quite recent date, and this is how the Babylonians and the Egyptians used their tables of multiplication, square roots, and sums of fractions. This is also the way in which they used the problem texts. A typical example, from an Egyptian papyrus of the middle of the second millennium, begins with a statement of the problem to be resolved:

> "A pyramid. Its side is 140 [cubits] and its slope is 5 palms 1 finger [per cubit]. What is its height?"

Here the data are presented in the form of concrete numbers rather than abstract variables, followed by a step-by-step solution with the answer at the end. Each step makes use of either the result of a preceding step or of one of the data given at the beginning of the problem.

No argument is given to justify the procedure nor is any explanation offered for its form. But even with the numerical values included, the nature of this form is quite clear. Thus the student would be able to solve any other problem of the same type. Moreover these problems are often grouped in such a way that the techniques learned can be immediately applied in other cases. The above problem, for example, follows a problem concerned with the determination of the slope of a pyramid given its length and height, and is followed in its turn by a problem involving the calculation of the slope of a cone.

However, not all mathematical problems were as practically oriented as this one. The main purpose of mathematical school exercises was to give the student scribe practice in the mathematical techniques used in solving problems. Technical drill, not direct application, was the main point. For this reason many of the apparently "practical" problems in these texts are far removed from real life: a Babylonian tablet proposes a problem in which a broken measuring rod is used to measure a field; an Egyptian document asks a scribe to calculate the original size of a herd of cattle based on the number of cows used to pay a cowherd's taxes...

The pedagogical purpose of all this is clear. Moreover, the structure of the problem and table texts permits an alternative approach to abstraction and generalization in mathematics. Rather than take the path of increasing symbolization marked by a hierarchy of "levels of generality", the Egyptian and Babylonian approach is to create a network of typical examples in which a new problem can be related—by a form of interpolation—to those already known.

Exactly the same approach can be seen at work in other areas of ancient thought such as medicine, divination and astrology. All these subjects were treated by the Egyptians and Mesopotamians as constituting a special domain of "rational practice".

No word for 'mathematician'

Ample documentary sources—accounting texts, lists of professions, references in literary and historical texts, even painting and sculpture—enable us to follow the professional life of the aspiring scribe once his education is complete. We

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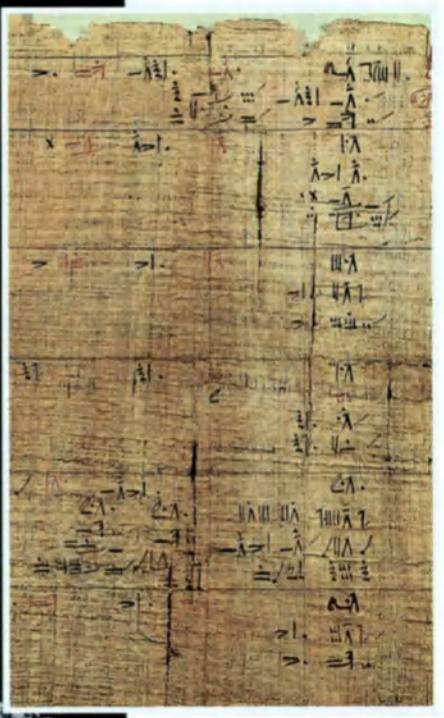


The Rhind Mathematical Papyrus, a scroll copied around 1650 BC from an earlier work, is the source of much information about ancient Egyptian mathematics. Detail, above, shows part of the table for doubling odd fractions.

Left, fragments of a Mesopotamian tablet from the beginning of the 2nd millennium BC.

will look in vain, though, for any mention of a "mathematician" in the modern sense of a person working within a recognized community of researchers on the properties of number and geometric figure. No word for "mathematician" exists in any of the ancient languages of Egypt and Mesopotamia.

There were two paths that a young scribe could follow. A few would become teachers of mathematics themselves, perhaps exploring further problems that could be presented to a new generation of schoolboys. In the course of time this did lead to a deepening and broadening of the mathematical techniques available to the two societies.

The graduate could also become an accountant—a calculator of work, rations, land and grain. Scribes are omnipresent and their industrious labourings are carefully depicted on Egyptian wall frescos and Assyrian palace reliefs, always hard at work on the business of their master, who might be either the state or a private landowner. They seem to have held privileged though subservient positions. Like their schoolteaching colleagues, they were not the movers of ancient society but they served those who were—and well enough to be immortalized on their patrons' walls as the visible symbols of that concentration of power and wealth they laboured so hard to calculate. ■

INDIA

Lilavati, gracious lady of arithmetic

BY FRANCIS ZIMMERMANN

Detail of a manuscript copy (date unknown) of a Sanskrit text composed in 750 AD and illustrated with geometric figures.

Opposite page, sundial at the open-air observatory at Jaipur (India), built in 1728.

DURING the eighth century AD, Arab scholars working in India on Sanskrit mathematical texts made two major discoveries which they developed and later transmitted to the Western world: the place-value notation of figures using the decimal system coupled with the "zero" concept, and a trigonometry which incorporated the use of sines.

It was not merely by chance that these major advances in the fields of writing, calculation and triangulation were made by Indian mathematicians; they all touched on matters of traditional interest in India, whose scholars had always shown a particular taste and talent for grammatical forms.

Mathematics, like all the other scientific disciplines in ancient India, was subject to the constraints and stylistic forms of the Sanskrit language as well as to the demands of verse since most scientific texts were written in verse form.

The great mathematical treatises, written in Sanskrit, usually by a Brahman (a member of the highest, priestly caste), consisted of a basic, often cryptic, text made up of *sutras* or aphorisms, or else of verses that were learned by heart. A stream of prose commentaries explain the full meaning of these ancient texts, confirming that they were aphoristic in nature and deliberately conceived as summaries of a master's teachings, expressed in such a way as to stick in the memory of his pupils.

Evidence of the early use of numbers, in the sense of graphic symbols, is to be found in inscriptions on stone or copper that have been studied by archaeologists—such as the numbers 4 and 6, for example, found in the Asoka inscriptions which date back to the third century BC. They are very rarely to be found, however, in proper mathematical texts. Arabic numerals, so called because they were made known to the rest of the world by Arab authors, are, in fact, of Indian origin. In general, however, they were rarely used in Sanskrit texts, in which numbers are written out in full or symbolized by alphabetical codes. To be more precise, we must distinguish clearly between the basic texts, generally written in verse, and the prose commentaries which alone provide us with information about the way in which numbers were written down during the actual process of calculation.

Numbers were set out vertically over several lines, or at least this is what emerges from a commentary written by Bhaskara the Elder in 629 AD on the *Aryabhatiya*. Unfortunately, the life-span of Indian manuscripts was short, averaging only about three centuries. Written on

A simple problem taken from the twelfth-century Indian mathematician Bhaskara's *Lilavati* ('Arithmetic')

अथ विश्लेषजात्युदाहरणम्—

पञ्चांशोऽलिकुलात्कदम्बवपगमत्रयंशः शिलीन्ध्रं तयो-
र्विश्लेषस्त्रिगुणो मृगाक्षि कुटजं दोलायमानोऽपरः ।
कान्ते केतकमालतीपरिमलप्राप्तैकचालप्रिया-
दूताहूत इतस्ततो भ्रमति खे भृङ्गोऽलिसंख्यां वद ॥

Example of the reduction of fractions to a common denominator:

One-fifth of a swarm of bees flew towards a lotus flower, one-third towards a banana tree. (A number equal to) three times the difference between the two (preceding figures), O my beauty with the eyes of a gazelle, flew towards a Codaga tree (whose bitter bark provides a substitute for quinine). Finally, one other bee, undecided, flew hither and thither equally attracted by the delicious perfume of the jasmine and the pandanus. Tell me, O charming one, how many bees were there?

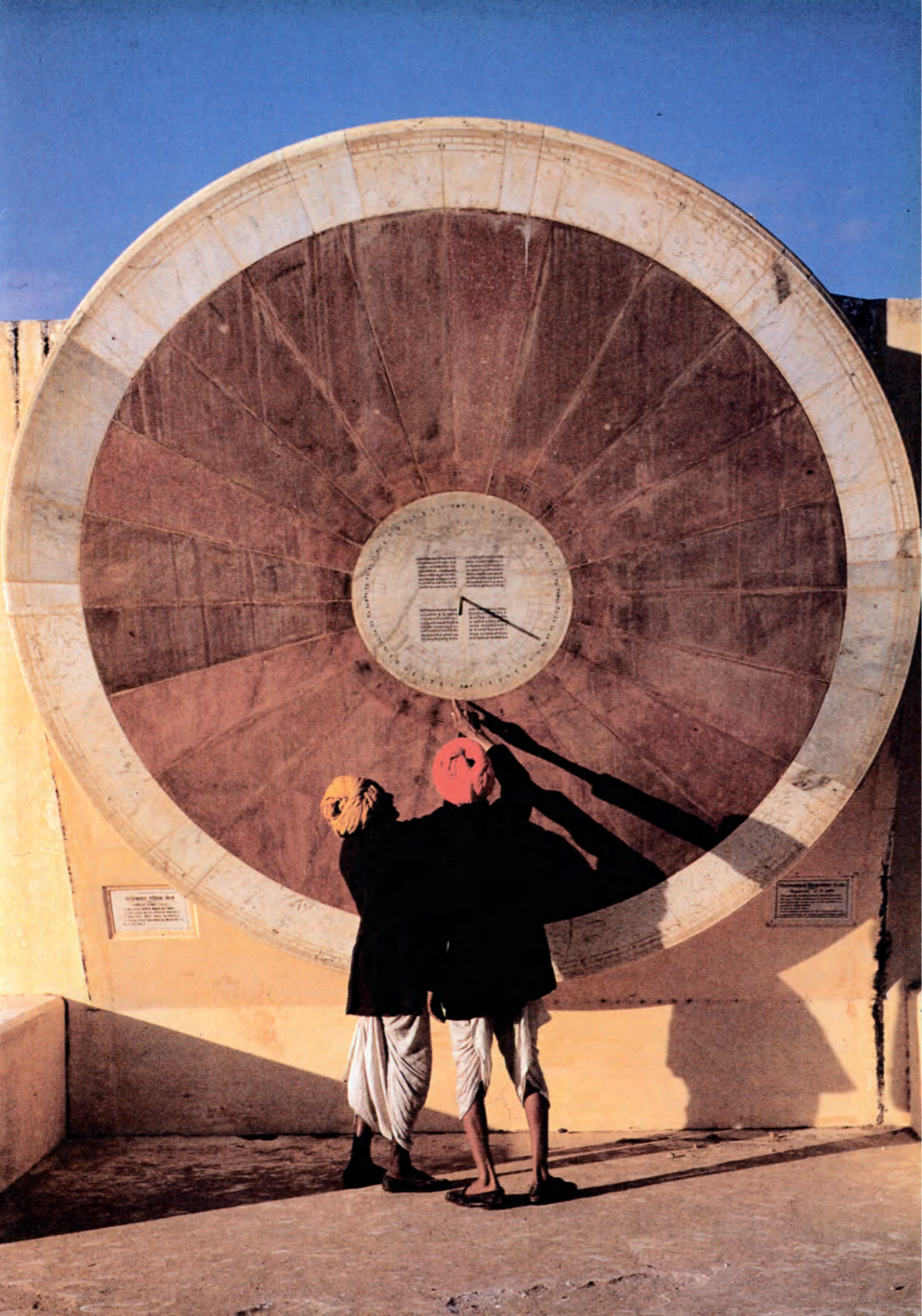
Let x = the number of bees

$$x = \frac{x}{5} + \frac{x}{3} + 3 \times \frac{(1 - 1)}{3\ \ 5} + 1$$

Reducing the fractions to a common denominator, we get:

$$x = \frac{3x}{15} + \frac{5x}{15} + 3 \times \frac{(5 - 3)}{15} + 1$$

$$x = 15$$



paper or on palm leaves, they were a prey to mildew and insects. The manuscripts of Bhaskara's commentary that have come down to us are modern copies and cannot be taken as evidence of the manner of writing of more ancient times. The twelfth-century Bakhsali manuscript, which is probably the oldest document in this sense, shows calculations in Arabic numerals spread over several lines and contained in cartouches, or boxes, within the body of a mathematical text written in Sanskrit.

The absence of graphic symbols and numbers in the aphorisms and verses of the classic mathematical texts does not mean that symbolism is totally excluded, but that the symbolism employed is grammatical or rhetorical in nature. Thanks to the virtually limitless synonymical possibilities of the Sanskrit language, numbers are represented by literary phrases and metaphors. Thus, *nayana* (eye) or *bahu* (arm) are the names of the number 2. *Agni* (fire) means 3 (by allusion to the three Vedic forms of ritual fire) and *adri* (mountain) means 7 (a reference to the seven mountains of India of Hindu religious geography). The Sanskrit words for "sky" or "space" stand for zero. The order of the figures making up a number is the reverse of modern numerical systems—for example, the number 23 would be written as *agninayana*.

This symbolism is useful for writing in verse form series of figures which today would be laid out in tabular form. In India, as elsewhere, the astronomical data in almanacs have for centuries been presented in columns of numbers. But this mode was an Arab invention and in the ancient Sanskrit texts numbers were presented in the form of a line or a verse of poetry.

Another form of numerical symbolism frequently used in astronomical and mathematical texts is based on the Sanskrit alphabet. Several such systems exist. The *katapayadi* system, which is widely used in southern India, makes it possible for very large numbers and trigonometric tables to be expressed in the form of mnemonic words, aphorisms or verses. The system is flexible enough to enable such numbers to be expressed in phrases which also have another meaning. For example, the priestly injunction *acaryavag abhedya*, which means literally "The word of the Master must not be betrayed", is coded writing of the number 1434160, a chronogram for the 1,434,160th day of the era of Kali, the day on which the philosopher Sankaracarya introduced certain reforms.

Did this poetic form of expression have an influence on mathematical reasoning? Was there some specific characteristic, something special in the way of thinking or the social status of Indian mathematicians which led them to shape their teachings in a literary mould?

There has never been a caste or even a real school of mathematicians in India. Mathematicians, if we class as such those who wrote or used Sanskrit texts dealing with geometry, arithmetic or algebra, worked in fairly close collaboration with experts in Vedic and Brahmanic ritual. Brahmans or members of a high caste and steeped in Sanskrit culture, they were classed among scientists as *jyotirvid*, or "experts on stars". Mathematical texts were usually inserted in treatises on astronomy, and trigonometry only really came into its own in the study of the angular distances between stars.

Like all the Brahman sciences (the *sastra)*,

Eleventh-century bronze effigy of the god Shiva as Vinadhara (master of sciences and arts). In Indian numerical symbolism, the Sanskrit word Rudrasya (the 5 faces of Siva) denotes the number 5.

The ratio of the circumference of a circle to its diameter (π)

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणां।
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥

caturadhikam satam astagunam dvāsastis tathā sahasrānām
ayutadvayaviskambhasyāsanno vrttaparināhah

"Add four to one hundred, multiply by eight, add 62,000. This will give you the approximate value (*asanna*) of the circumference of a circle with a diameter of two myriads."

This verse, written by the sixth-century Indian mathematician Aryabhata, gives the oldest known formulation of the approximate value of the ratio that later became known as π:

$$\frac{\text{circumference}}{\text{diameter}} = \frac{62832}{20000} = 3.1416$$

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mathematics had been developed primarily for religious purposes, as an aid to the proper performance of ritual. We know nothing of the life of the great Indian mathematicians, but we can picture fairly accurately the ritualistic and scholastic setting within which they worked, so heavily coloured by it is the style of Sanskrit texts. After the pupils had memorized a text by repeating it word for word, time and again until it was "clasped to their bosoms" (until they had learned it by heart), the teacher would provide them, orally, with the illustrations, demonstrations and calculations that the text concealed. This was the key that opened up the paths of knowledge and an instrument of spiritual fulfilment.

The *Lilavati,* a text by the twelfth-century mathematician Bhaskara the Learned, was traditionally used for this purpose for arithmetic and this explains why it terminates with a verse with a double meaning in which Bhaskara compares his *Lilavati*, his "gracious one" (it also means arithmetic), to a woman endowed with all the graces of the *jati* (a word which means both of noble lineage and, in a technical sense, the reduction of fractions to a common denominator). "Joy and happiness in this world shall continually increase for those who hold her *kanthasakta*, close in their arms or clasped to their bosoms (learned by heart by repetition)."

From ritual geometry to Bhaskara's treatise

The oldest of these texts that have come down to us are the *sulbasutras,* "maxims concerning measuring cords", which are believed to have been written between the fifth and the first century BC. They are treatises which lay down the rules for building altars used for Vedic ritual sacrifices and made of bricks laid in accordance with symbolic forms. The geometric constructions which are taught in these treatises are based on knowledge of several special cases of right-angled triangles (for example, with sides measuring 3-4-5, or 5-12-13, 7-24-25, etc.) and on the general rule according to which "the diagonal of a rectangle produces (by the construction on it of a square) the equivalent of the product of both the rectangle's length and breadth; and the diagonal of a square produces (by the construction on it of a square) twice its own area." Here, however, the rule is not expressed as a theorem, but as a maxim, as a formula to ensure the proper functioning of ritual and as a building guideline. The word *sutra* itself, which at first meant "aphoristic in style", came, in the later treatises, to mean a "rule" in the technical sense of a rule for building.

There are no theorems in Indian mathematics, only rules based on reasoning which starts from an intuitive point of departure. The rules, aphorisms and the mnemonic verses of the basic texts are not the outcome of a demonstration but rather guidelines for a geometric construction to be carried out by the reader or the commentator. Even in algebra, the typical line of reasoning links areas to the products of factors and implies the construction of a geometric figure.

Nanda, the adoptive father of Krishna, consults an astrologer. Miniature of the Kangra school (late 18th century).

It has often been said that the Indians were algebraists rather than geometers yet, in fact, throughout all the commentaries on the teachings of Aryabhata (sixth century), Brahmagupta (seventh century) and Bhaskara (twelfth century), geometry was the source of the practical applications of the rules of arithmetic and algebra. A geometric space and a numerical ensemble were taken together as two facets of the same reality. The algebraic solution was grafted on to the geometric construction. To demonstrate was to display the solution, to render it intuitively manifest. As one commentator said: "A demonstration by quantities should be made for the benefit of those who do not understand the demonstration by areas." Thus, in Indian mathematics, to reason is to explain an intuition. ■

π in the sky

BY JEAN-CLAUDE MARTZLOFF

PREDICTION OF CELESTIAL PHENOMENA WAS A MAINSPRING OF EARLY CHINESE MATHEMATICS

"CHINESE mathematics", defined by the Chinese themselves in ancient times as the "art of calculation" *(suan chu)*, covers a vast range of practices and currents of thought that grew up in China between the first millennium BC and the fall of the Manchu dynasty in 1911. After that date Chinese mathematics became westernized and the traditional practices were virtually impenetrable to those without specialized training.

Divination, astronomy and mathematics

Although writing already played an important role in China at the time of the composition of the earliest canonical writings (the "classics" which were always so essential in the training of intellectual élites), mathematics was not then considered to be a body of knowledge that it was necessary to record in specialized texts.

It seems, however, that mathematics was instrumental in the emergence of what the sinologist L. Vandermeersch has judiciously called "divinatory rationality". Beginning with oracles in the form of tortoise shells, animal bones or the leaves of the Chinese yarrow, the prognostics made using this form of divination were based on the interpretation of all manner of natural signs, especially meteorological and astronomical phenomena such as rainbows, halos, winds, meteors, positions and conjunctions of stars, eclipses and sunspots. This magical worldview did not however exclude recourse to purely rational investigations. The diviners attempted with some success to fit all their observations into numerical and arithmetical schemes devised for the recording of memorable past phenomena and the prediction of certain recurrent events. Some predictions of periodically recurring celestial

1,420 stars are depicted on this polychrome sky map made in 1453. The map forms part of a panelled ceiling in Beijing's Longfu temple.

phenomena began to be confirmed by observation, and led to the birth of the calendar and mathematical astronomy.

Each successive dynasty in China made its mark by introducing a new system for computing the calendar so that the historical events recorded by annalists could be reassessed and future events predicted. The ruling class therefore needed qualified personnel who specialized in calendrical and astronomical calculations. A body of imperial chronologers thus gradually came into being, serving both as historian-annalists and astronomer-calendrists.

The continuing requirements of Chinese dynasties down the ages explain why the search for appropriate methods of predicting the most visible celestial phenomena (conjunctions of stars, occultations, eclipses) was foremost in the minds of mathematicians.

However, as the astronomer-calendrists of imperial China usually had a lowly social status, and as their knowledge was handed down from father to son, their activities were often denigrated and looked upon simply as a static way of upholding tradition.

The overwhelmingly dominant factor in the history of Chinese mathematical astronomy is the extraordinary durability of conflicts between rival schools. From the beginning of our era to the sixteenth century, the Chinese calendar underwent no less than fifty reforms. However, these conflicts turned out to be more constructive than destructive in that they were almost always settled by the concordance between observed reality and the predictions.

Unfortunately, very few works devoted exclusively to mathematical astronomy have survived. Monographs written by non-specialists, abridged versions of which were included in the annals of successive dynasties, are practically all that remain.

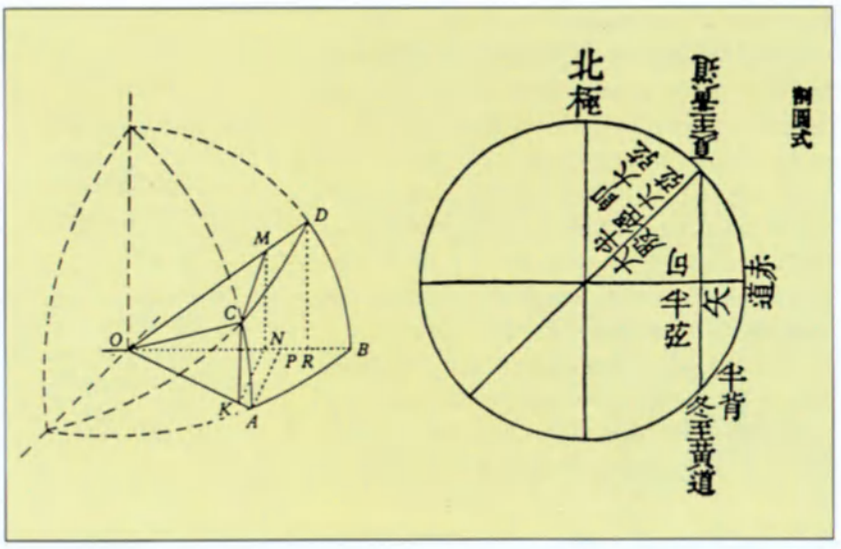
The legendary emperor Yao requests the astronomers Hsi and Ho to establish the calendar and pay tribute to the heavenly bodies. Engraving from the late Manchu period (1905).

The social background of Chinese mathematics

During the Han dynasty (206 BC to 220 AD), another branch of mathematics appeared, this time recorded in specialized manuals. Collections of problems and their solutions were organized in chapters according to their practical applications. The descriptions in these texts are often so detailed and realistic that it is almost possible to reconstruct from them entire chapters of Chinese social and economic life at a given period. Whether they are about tax collection, corvée labour, weights and measures, currency, the construction of dykes or canals, the management of manpower, land or river transport, policing or military logistics, no practical detail was neglected. Generations of "civil servant-mathematicians" were trained for the imperial bureaucracy by the study of such manuals.

The Tang dynasty (618 to 907 AD) set up an examination system whose syllabus included mathematics as well as letters. Although least importance was accorded to mathematics, study of the subject at the *Guozijian*, the "College of the Sons of the State", lasted seven years and was based on a classic textbook, "The Ten Mathematical Manuals" *(Suanjing shi shu)*. In 1084, under the Southern Sung dynasty, this text was printed. Yet after 1230 mathematics was banished from the examination syllabus for good, in favour of literary subjects.

Paradoxically, the high-water marks of mathematics are not to be found in the short periods when the teaching of the subject was institutionalized. On the contrary, the most remarkable advances took place when the empire



Diagrams illustrating problems in spherical trigonometry examined by the astronomer-mathematicians Kuo Shou Ching (1276, left) and Hsing Yün Lu (1600, right).

was divided, in time of war and bureaucratic breakdown. During the Warring States period (453 to 222 BC), the Moists (followers of Mo-tzu who challenged the dominant Confucian ideology) developed the rudiments of theoretical geometry, but their method had little influence on the main current of mathematical thought. The greatest mathematician of ancient China, the details of whose life are unfortunately unknown, was Liu Hui, who elaborated several mathematical theorems at the time when China was split into three kingdoms (220 to 265 AD). During the Mongol conquest in the thirteenth century, a wealth of original achievements took root, flowered, and were quickly consigned to oblivion.

The last and greatest of the Sung mathematicians was Zhu Shijie (fl. around 1300), a wandering scholar who travelled far and wide teaching the results of his research. Also during these

Lid of a coffer dating from around the beginning of the Warring States period (c. 433 BC). The Great Bear (at centre) is surrounded by the names and symbols of the 28 lunar mansions.

troubled times, the hermit Li Zhi (1192-1279) organized a private group devoted to the study of the mysteries of the world and of numbers, which led to the development of algebra in China. And in the nineteenth century, shortly after the Opium War, when the Chinese empire was overtaken by major disasters, Li Shanlan (1811-1882) failed his literature examinations and dedicated himself to the joys of mathematics. He discovered an astonishing series of perfectly correct formulae which were to baffle even a twentieth-century mathematician of the calibre of Paul Turan of Hungary.

The reasons behind these sudden and short-

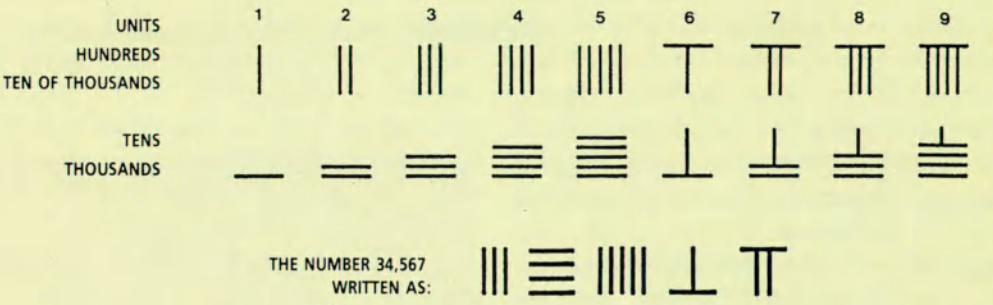
lived flashes of brilliance are poorly understood. Perhaps political anarchy freed the intellectual élite from the obligation of preparing the sterile mandarinate examinations. Confucianism, the dominant ideology, relegated mathematics to a minor role. Those who were attracted by the subject could therefore give free rein to their intellectual curiosity without great risk. Even in stable times, however, many Confucian scholars used their leisure hours to study mathematics. From the eighteenth century onwards, more and more philologists and specialists in textual criticism dreamed of turning mathematics into a branch of history. Some of them tried their best to use mathematical astronomy to authenticate the classics. They wanted to define by calculation the historical reality of natural events recorded in ancient documents (especially eclipses of the Moon or the Sun). Others enthusiastically studied "concrete sciences" such as economics, hydraulics, civil engineering or architecture. Out of these excursions along lonely and difficult paths there sometimes grew a taste for pure mathematics, independent of any particular application.

Contact with other cultures is also of importance in the history of Chinese mathematics: with India during the first millennium AD as Buddhism was being preached abroad; with the Arab

Gilt bronze equatorial Moon dial. Qing Dynasty (1744).

COUNTING-ROD NUMERALS OF ANCIENT CHINA

The use of counting-rod numerals, which may have originated with the arrangement of actual rods laid out on a flat board, developed in China around the 3rd century BC. Units were noted vertically, tens horizontally. The value of the signs was determined by their position:



and Persian peoples at the time of the Mongol expansion; and with European missionaries from the late sixteenth century onwards. Although many similarities can be seen between mathematics in China and in other cultures before 1600, it is unclear whether this is a sign of actual influence or simply of parallel development.

For example, the Chinese symbol for zero, which was written as a small circle as it is today, appeared for the first time around 1200 in astronomical tables and could have been of Indian origin. A number of ancient and medieval mathematical games, whether from the Hellenistic or the Arab world, India, Europe or China, have a surprising family likeness. Very similar mathematical techniques existed at the same period in China and in Greece. To cite but two examples, Euclid and Liu Hui both discovered methods of determining the volume of a pyramid; Archimedes and Liu Hui found the volume of a solid by the intersection of two orthogonal cylinders.

Even if these parallels do indicate the transmission of influences, Chinese mathematics had an internal coherence and developed independently.

Geometry without parallel

In traditional Chinese mathematics there is no geometrical reasoning based on axioms, postulates, definitions and theorems. There are no absolute truths in the manner of Euclid, but only relative and provisional truths. In geometry, there are no angles nor even any parallel lines, but only lengths, areas and volumes. Neither is there any algebra in the Arab style, nor the search for the roots of equations using radicals or intersecting curves. And there is no "rhetorical" algebra enshrined in prose or verse.

Yet Chinese mathematics is based not on purely empirical procedures but on heuristic principles. The emphasis is on the answer itself, rather than on explaining the intermediate stages in getting there, which are taken for granted. For example, one of the cardinal rules of Chinese geometry stipulates that the surface or the volume of a figure remain the same after it has been broken up and re-assembled, even though the number of the fragments is potentially infinite. This type of principle does not at all exclude the recourse to an axiomatic system, but it is true that Chinese geometric figures were not usually abstract ideals. They were, rather, like tangible pieces of a puzzle that can be distinguished by their colour and manipulated at will. Chinese geometry basically relies on a shrewd and meticulous examination of a problem to reveal certain results. This method is not only important in the calculation of areas and volumes, but also in bringing out certain properties of the right-angled triangle, in calculating the total of a series, in resolving equations or systems of equations, and in the visual proof of mathemathical identities.

What is more, Chinese geometry sees nothing wrong (as Euclid would have done) in resorting to calculations or any other method that may be useful to solve a given problem. This attitude shows Taoist influence, and Chinese mathematicians of the third to the fifth centuries AD declared their boundless admiration for Zhuangzi, the father of philosophical Taoism. This eccentric sage had rejected the use of language as a notably effective way of reaching the truth, on the grounds that the fallacious arguments of the sophists showed its limits.

He had concluded that all attempts to reach the truth founded on discursive argument, that is to say on a method capable of engendering manifestly false conclusions, would be doomed to failure. Hence the tendency of Chinese mathematicians, influenced by Taoism, to place only limited faith in the power of language. On the other hand they tended to make use of all the means at their disposal, without ever ignoring the evidence of their senses. This is why they were

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so fond of calculations and all kinds of manipulations, so long as they could avoid putting them into words. Only as a last resort did they have recourse to discursive arguments.

How did mathematicians apparently so obsessed by the concrete have produced sophisticated results? In fact the practical nature of Chinese mathematics does not imply any lack of abstraction in their thinking. On the contrary, some of the results obtained from manipulating their puzzles presuppose great ingenuity and a remarkable capacity for abstraction.

Moreover, Chinese mathematicians often deliberately distorted reality, as they could not easily teach their subject using as examples the increasingly complex problems of daily life. This is why many Chinese mathematical problems conceal purely fictitious situations behind a veneer of practical reality: values which are impossibly large or small, or that make no sense (fractions of men); data arranged in an arbitrary manner, as when areas are added up with volumes and prices; inversion of known and unknown values as when calculating the dimensions of objects from their volume, or capital from the interest gained, or quantities of goods from the share received by each person. Obviously such methods opened up very much more interesting situations from the mathematical point of view.

Chinese algebra developed on this "fictional territory". In the oldest manuals are many calculatory formulae for resolving limited classes of problems. In extreme instances, each problem constitutes a specific case. Later, generalized procedures appear by which an increasingly wide range of problems could be solved, so fictitious situations were no longer required.

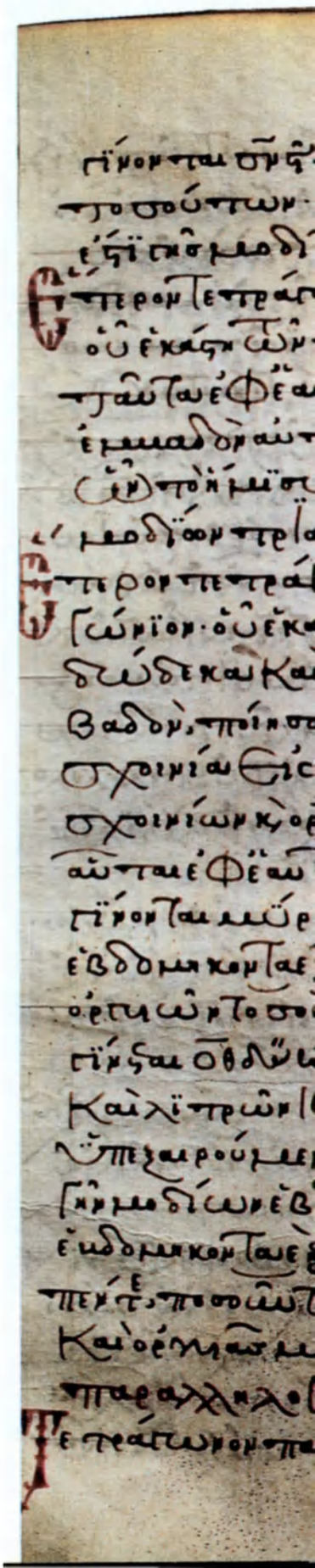
All this could not have taken place without Chinese computational methods, which were based on mechanical aids. The abacus is perhaps the best known of these, but it appeared at a late date (around the fifteenth century). Chinese mathematicians relied above all on the manipulation of counting-rods *(chousuan),* which were arranged to represent the various coefficients of numerical equations. Mathematical problems transferred to arrangements of counting-rods were, so to speak, divorced from their practical context and taken into the realm of abstraction.

This method of calculation by tabulation is generally known as *fangcheng (fang* meaning "square" or "rectangle" and *cheng* meaning "divide up") and involves arranging the counting-rods to form a square or a rectangle (a matrix). Two types of rod were used, red and black or positive and negative, representing the complementary forces which controlled the Chinese universe, *yin* and *yang.*

Here we have an "instrumental" algebra which dispenses with all discursive methods of argument. This is its strength, but also its weakness, as during the manipulations of the counting-rods, calculations vanish without trace as soon as they are completed. This "art of the rods" is like the art of a musician who plays without looking at the score, and it is no coincidence that some Chinese mathematicians explicitly compare mathematics with music.

Calculations and manipulations are the basic ingredients of a form of mathematics which has never felt itself to be bound by any dogma, and which at various phases of its history has syncretized many elements from other cultures. ■

The gelosia or grating method of multiplication as shown in a Chinese book of 1593.

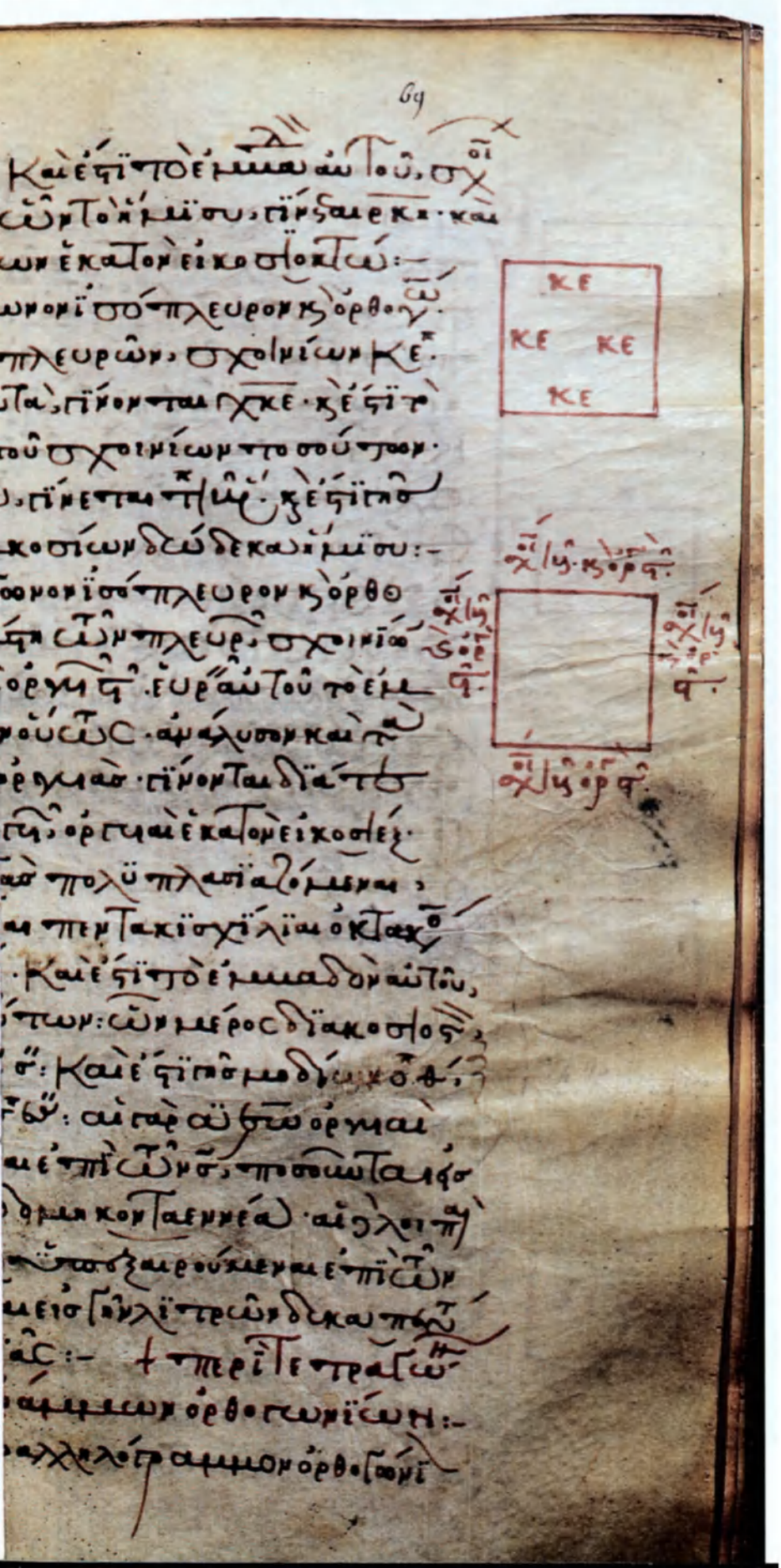


Right, detail from a 12th-century Greek manuscript of Euclid's Elements.

ANCIENT GREECE

The Odyssey of reason

BY BERNARD VITRAC



IT is scarcely necessary to recall the importance of the role played by the mathematics of ancient Greece in the development of this discipline in the West. The very words "mathematics" and "mathematician", or their equivalents in most European languages, are derived from the Greek word meaning "to know" or "to learn". Before the classical era, however, when it took on the specialized meaning that it has today, the Greek word *mathema* meant "that which is taught", in other words all branches of knowledge.

The teaching of mathematics

We know very little about the teaching of mathematics in ancient Greece. It seems that some philosophical schools played a major part in the training of mathematicians, at a time when intellectual specialization was still the exception.

During the classical era there is reference to the existence of "scientific" schools, such as those at Chios and Cyzicus. However, we do not know whether these schools offered general or specialized education, or whether they were anything more than groups of scholars gathered around a famous teacher.

As with medicine—for the existence of schools of which there is earlier and more solid evidence—it seems that family background may have had an influence on professional training. We have few biographical details about the early mathematicians, but we do know that Archimedes was the son of an astronomer, that Hypsicles' father was a mathematician, that the geometers Menaechmus and Dinostratus were brothers and that Hypatia, the only Greek woman mathematician about whom we know anything much, was the daughter of the mathematician Theon of Alexandria.

In the centralized states of the Near East of ancient times, the need to train a class of scribes and functionaries was recognized at an early stage.. The Greece of the classical era, however, consisted of small, independent city-states, continually warring among themselves, or of loose, clan-like associations, and had no need of organized educational systems such as those of Egypt, Babylon or China.

Although trade, land surveying and navigation demanded a certain minimum knowledge of mathematics, and although elementary calculation was taught in schools, the Greek city-state paid scant attention to the intellectual and

technological education of children and youths. Schools, some of which were to achieve considerable renown, were the fruit of private initiatives. The two great Athenian masters of the beginning of the fourth century BC, Isocrates and Plato, each founded his own educational establishment—Isocrates a school of rhetoric and Plato a school of philosophy.

Both of them regarded mathematics as an indispensable tool for intellectual development and valued the "mental gymnastics" and concentration this discipline required. But their approaches to mathematics differed. Isocrates thought that mathematics, like the adversarial debates that young people were so fond of, should develop "well-formed minds", even if the subject matter was of little value to the citizen, who, ideally, would devote himself to political life. Plato, however, whilst recognizing its propaedeutic value, saw mathematics as a preliminary to the study of philosophy—in other words, of Platonic idealism—and also as a method of selection, since the mathematics and the philosophy he taught together constituted a form of intellectual asceticism essential to his project for political reform.

Pythagoras (6th century BC) as depicted in a 16th-century engraving.

The third and second centuries BC witnessed a considerable development in the mathematical sciences. Most of the works that have come down from this period are those of mathematicians more or less closely linked to Alexandria, the capital of the Greek Ptolemaic dynasty which ruled Egypt from 306 to 31 BC. It is known that the Ptolemies adopted a broader policy of state patronage—previously restricted to a few individuals, often poets—by creating a number of institutions, the best known of which were the Alexandria Library and Museum. These new institutions undoubtedly gave an impetus to literary studies. That they had an effect on scientific development is probable but less certain, but the favourable climate they created could only be beneficial to it.

However, we do not even know if the great scholars of the time, whose presence in Alexandria is well documented—Herophilus of Chalcedony, Euclid, Strato of Lampsacus, Aristarchus of Samos, Eratosthenes of Cyrene, Apollonius of Perga—taught their disciples or gave lessons or even lectures, whether under the auspices of the Museum or privately. Indeed, it was only in the time of the Roman Empire that the Museum was to operate as a university, to be copied in this at Ephesus, Athens, Smyrna and Aegina.

Alongside pure mathematics, in the Greek tradition, there was also a corpus of mathematical texts that might be termed "calculative"—similar to those to be found in Egyptian, Babylonian or Chinese mathematical writings. For example, a body of comparatively late mathematical texts, attributed to Hero of Alexandria, was compiled and used right up to the Byzantine era, probably for the training of technicians. As in Babylonian and Egyptian texts, the problems set refer explicitly to a factual situation, even if this is merely a teaching device.

Nothing like this is to be found in the classic treatises of Euclid, Archimedes or Apollonius, who showed scant regard for practical applications. Euclid's exposition of the theory of numbers even manages to avoid reference to numerical examples. The works that have survived seem to indicate a strict separation between pure research and practical applications. Yet even though the two are so clearly distinguished, the same authors gave equal attention to both "pure" and "applied" mathematics.

That section of Greek mathematics which, by convention, we have designated as "pure" had four basic characteristics:

Thales of Miletus (late 7th-early 6th century BC). 17th-century engraving.

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Deductive presentation: the classic treatises, such as Euclid's *Elements*, are organized on deductive lines. The results are proved by demonstration, either from results previously achieved, or on the basis of principles established from the start. This might be described as a semi-axiomatic approach, which emphasized the logical, ineluctable aspect of mathematics. It should, however, be noted that this is often difficult to dissociate from the rhetorical aspect, which had the advantage of compelling the attention of the student, aimed at psychological and pedagogical effectiveness, and concentrated on the necessary objective structure of reasoning.

Geometrical orientation: even when they were concerned with the theory of numbers, statics or astronomy, those treatises that were demonstrative in style were basically geometric in their approach. Ancient mathematicians had introduced various symbols to designate numbers and fractions and also employed certain abbreviations, although these did not constitute a complete set of algebraic notations such as we use today.

However, it was in the use of geometrical figures that the Greeks went farthest in their experimentation with the use of "representative" symbols. The possibility of breaking figures up into elements, of establishing rules of construction that could be quoted, and the revelation of

properties which seemed already to be "present" in the figures, all these features were perfectly suited to the method of deductive exposition.

The ideal of disinterested science: the love of knowledge itself was the driving force behind the study of mathematics.

Mathematics and philosophy: the development of mathematics was contemporaneous with the development of philosophy.

Philosophers and mathematics

These developments in mathematics were paralleled by the opening of a methodological and ideological debate on the sciences. The classification of the sciences proposed by the Greek astronomer-mathematician Geminus (see box page 34) affords a good example. His classification assumed the existence of a body of scientific knowledge already fairly well developed and very varied. It adopted a differentiation between the sciences which favoured abstract study, which it detached from possible practical applications.

According to Aristotle, mathematics was the study of properties that could be "abstracted" from the objects of the physical world. Furthermore, like all the demonstrative sciences, it was based on principles, so that one science could presuppose another and, in Aristotle's words: "be subordinate to another." Optics, for example, was "subordinate" to geometry. Thus there was a logical hierarchy of the sciences. This had to be distinguished from the division, commonly stressed by Greek authors, between "practical" and "disinterested" mathematics. Only the latter, according to Aristotle, was worthy of inclusion in a liberal education. "To be free" was an end in itself.

The art of embellishment was superior to practical technology, but disinterested science, an end in itself, was the supreme activity. For Plato, the mathematics developed by the Barbarians, prestigious though their civilizations might have been, ranked as mere arts, since they were encumbered by the constraints of necessity. Greek philosophers thus based their argument on a mixture of methodological and ideological considerations.

Though treatises on optics and astronomy adopted the style of geometrical exposition, the deductive method being particularly suited to the elimination of anything that smacked of the "tangible" and the "practical" (see box page 34), it is not easy to see how Greek mathematicians managed to identify themselves with this way of looking at their activities. Moreover, though it is tempting to do so, we should refrain from confusing the modern distinction between "pure" and "applied" mathematics with the ancient division between "intelligible" and "tangible", since the concepts do not coincide.

Evocation of the ideal of "disinterested science"

leads on to the problem of what motivation there was for the advancement of mathematics. A distinction has to be made between external forces and what might be described as internal influences. Among the former, an important role was played by optics and astronomy, which we classify as physics but which the ancients included in mathematics. To these should be added statics, the science of equilibria.

What is known of the internal influences? Perhaps they are to be found in the prefaces with which, from Archimedes onwards, author/mathematicians seem to have introduced their works. What emerges from these prefaces is that far from being the product of psychological characteristics peculiar to the Greek mind, disinterested research presupposes the existence of a community of mathematicians all abiding by certain standards.

First and foremost, these mathematicians felt under no obligation to justify the fact that they were practitioners of science for its own sake; that went without saying. At the most they might acknowledge that they had opted for mathematics rather than for "physics" or "theology" because it was a more certain and rigorous discipline that was "stable" (unlike physics), yet nevertheless "accessible" (unlike theology).

PRECEDING PAGE

The Death of Archimedes. In this mosaic found at Herculaneum, the Greek mathematician is shown drawing geometric figures when surprised by a soldier of the Roman army which took Syracuse in 212 BC. When called on to give himself up, Archimedes is said to have replied, "Do not disturb my circles".

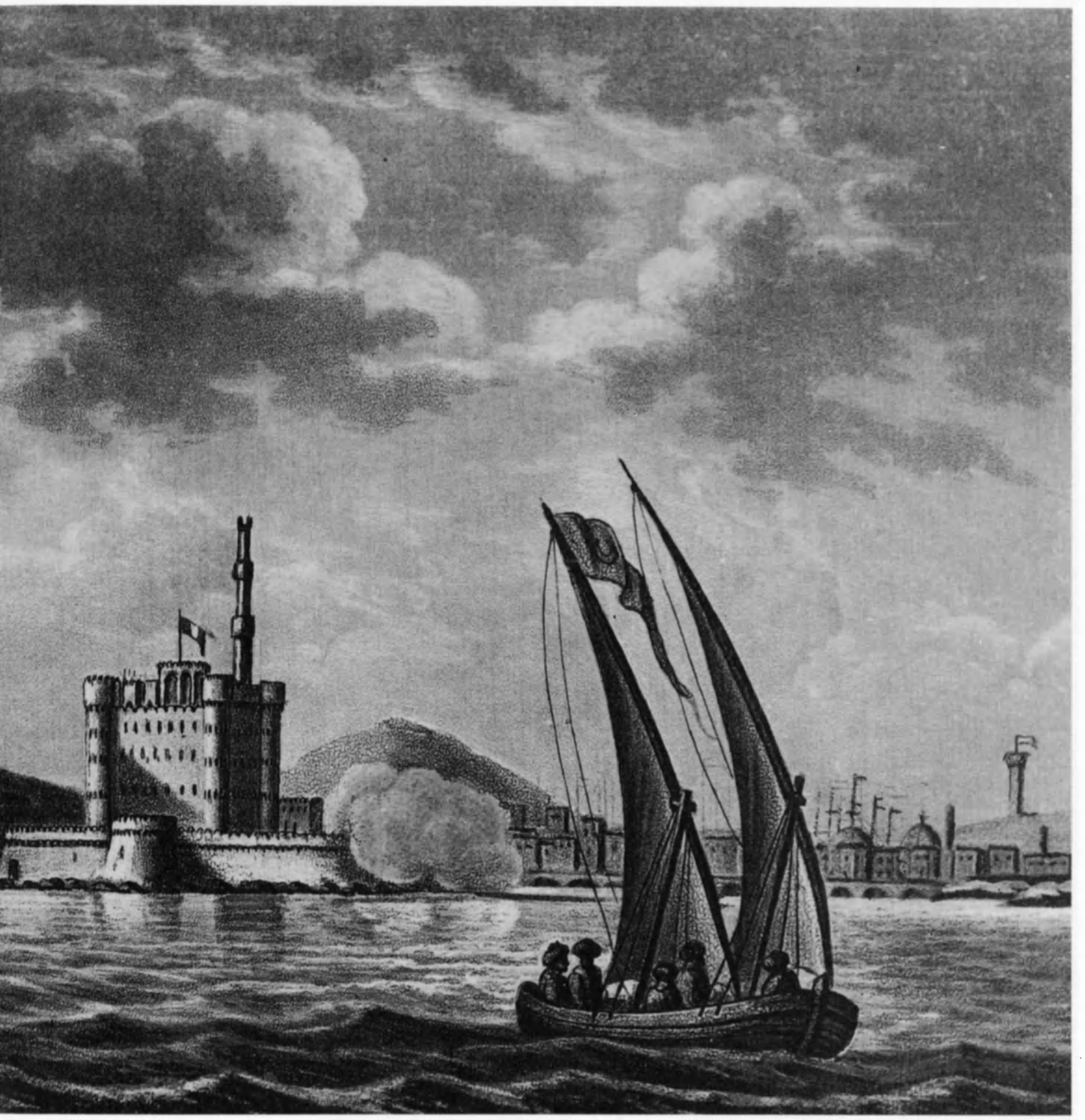
Geminus' classification of *Mathemata*

Others, among them Geminus, claim that mathematics should be sub-divided in another fashion. On one side they place tangible things, or anything connected with them. No doubt they would call intelligible the subjects of contemplation that the soul elaborates within itself, cutting itself off from things material. They rank arithmetic and geometry as the two first and most important sectors of that form of mathematics that deals with that which is intelligible. With regard to the mathematics which covers things tangible, they designate six sectors: mechanics, astronomy, optics, geodesy, canonics and logistics.

On the other hand, for them, unlike others, tactics is not worthy to be considered a part of mathematics, even though at times it involves logistics, as in the enumeration of troops, and geodesy, as in the surveying and the division of land. Even less do they consider either history or medicine to be part of mathematics, despite the fact that the authors of historical works often refer to mathematical theorems to indicate climatic conditions or to calculate the size of a town, its diameter and its perimeter, and that physicians use these procedures to throw light on many matters within their competence. The value of astronomy to medicine has been made perfectly clear by Hippocrates and all those who studied the seasons and regions.

Proclus, Commentary on Book I of Euclid's *Elements*, 38.

During the Hellenistic era, mathematicians formed an "international" community whose members were scattered around the shores of the Mediterranean (Greece, Asia Minor, Egypt and Sicily). They maintained personal contacts, either by paying each other visits or by circulating their latest works. Above all, they would set each other problems, solve those that were sent to them, or criticize solutions submitted by others which they considered to be flawed. By this means some of them acquired a commonly recognized status of authority. Publications were submitted to their judgement which they in turn circulated to others considered worthy to receive them. Among them



The famous lighthouse known as the Pharos of Alexandria is pictured in this engraving made in 1801.

also were a few impostors who would be unmasked by the expedient of setting them impossible problems which they would claim to have solved. These contacts, of course, remained personal and were not made within the framework of any form of institution in the modern sense of the term.

The ideal of "disinterested mathematics" seems, in short, to have been linked to the existence of a group within which problems of rivalry and competition were not altogether unlike those characteristic of the modern scientific community. The comparison should not be pressed too far. There is, for example, a marked difference in scale between the ancient and the modern communities. The number of scientists, and in particular of mathematicians, in the Hellenistic era certainly never exceeded a few hundred. Likewise, there can be no doubt that, in the absence of any true form of institution, the community they formed was a very precarious one. From the Roman era onwards, the best authors (Ptolemy, Pappus) were concerned, so it would seem, only with perfecting results already achieved. The rivalry and the search for new theories that marked the preceding era had apparently disappeared. ■

THE ARAB WORLD

Where geometry and

algebra intersect

Historian of science Roshdi Rashed on the Arab contribution to the history of mathematics

The city of Baghdad as depicted in a Turkish miniature of 1534.

How would you describe the beginnings of Arab mathematics?

Arab research into mathematics seems to have begun early in the ninth century when, in Baghdad, the movement to translate the great Greek authors into Arabic was at its height. For instance, Al-Hajjaj ibn Matar made two translations of Euclid's *Elements* and Ptolemy's *Almagest,* and Hilal ibn Hilal al-Himsi translated the first four volumes of Apollonius of Perga's *Conics.* Several works by Archimedes, Pappus and Diophantus, among others, were also translated in the same century.

This major undertaking was noteworthy for two very important features, in that the translations were made by leading mathematicians and were inspired by the most advanced research of the time. For example, volumes V to VII of Apollonius' *Conics* were translated by the great mathematician Thabit ibn Qurrah, who died in the year 901. Moreover, all the indications are that Qusta ibn Luqa was prompted to translate the *Arithmetica* of Diophantus around 870 on account of studies already being carried out on indeterminate analysis.[1]

Many other examples could be cited to demonstrate the close links between such translations and the innovative research already being done at this high point in the dissemination of Hellenistic mathematics in Arabic.

Al-Khwarizmi is the best known of the Arab mathematicians. What was his contribution to mathematical thinking?

It was at this time, in the ninth century, at the Baghdad Academy, or "House of Wisdom" as it was known, that al-Khwarizmi wrote a work that represented a new departure in terms of both content and style. This was the *Kitab al-jabr wa'l muqabalah* ("The Book of Integration and Equation"),[2] from which algebra emerged for the first time as a separate and independent mathematical discipline. This was a crucial development, and was recognized as such by al-Khwarizmi's contemporaries, as much for its new mathematical style as for the nature of its subject-matter and above all for the rich promise it held out for further advances.

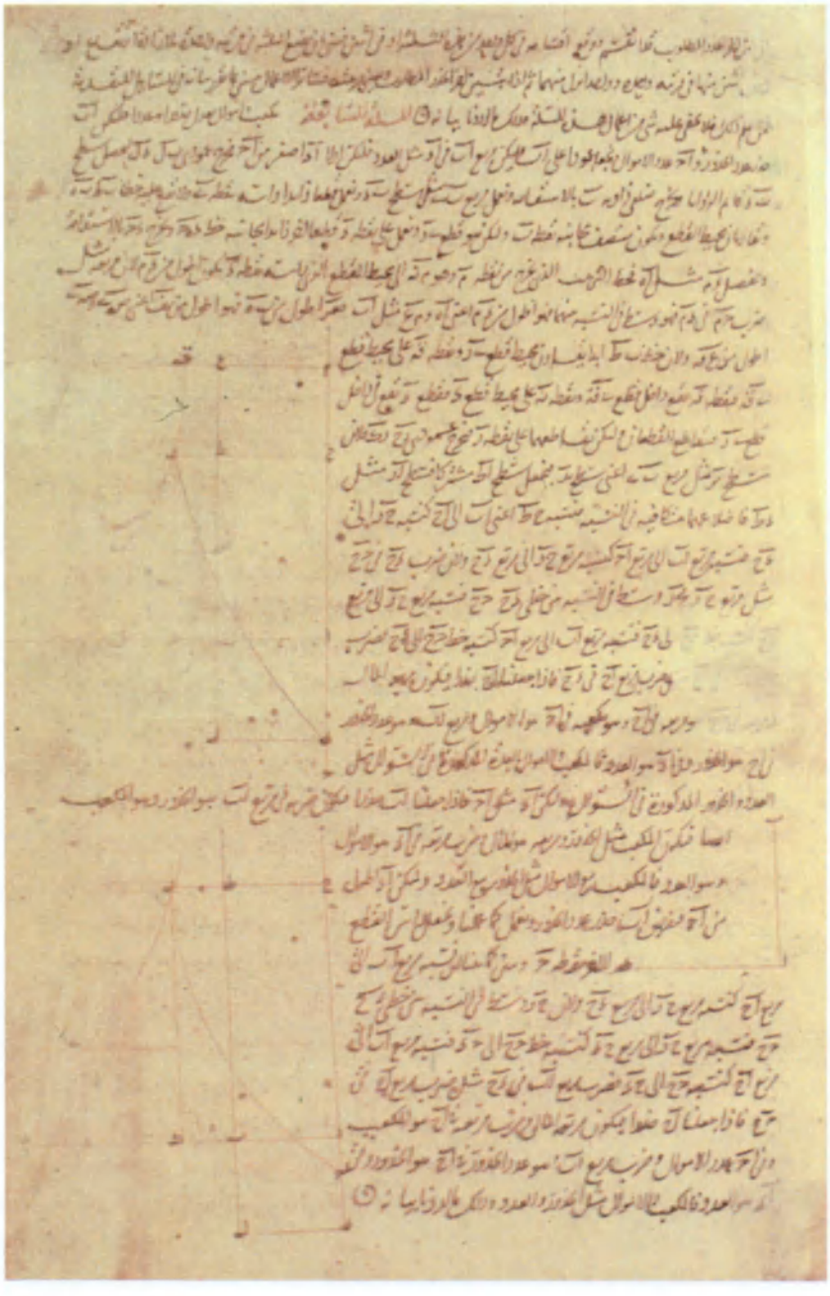
In terms of style, it was both algorithmic, in that the author set out a series of computational procedures, and demonstrative. A new mathematics had to be devised that was general enough to be capable of handling different types of formulation while existing independently of those formulations. In al-Khwarizmi's work, an algebraic

expression could refer to a number or to an irrational quantity or to a geometrical magnitude. This new mathematics and its combination of the demonstrative and applied approaches was a striking innovation for the thinkers of the time.

The novelty of the conception and style of al-Khwarizmi's algebra, which did not hark back to any previously known tradition, cannot be over-emphasized. The new algebra already afforded a glimpse of the enormous potential for applying one mathematical discipline to another that lay ahead from the ninth century onwards. In other words, while algebra, by virtue of its widespread scope and the new concepts it introduced, made such applications possible, their number and variety were thereafter constantly to change the face of mathematics.

Al-Khwarizmi's successors increasingly applied arithmetic to algebra and vice versa, arithmetic and algebra to trigonometry, algebra to the Euclidian theory of numbers, and algebra to geometry and vice versa. All these applications paved the way for new disciplines or at least for new chapters in the history of mathematics.

Detail from a treatise on equations by Sharif al-Din al-Tusi (late 12th-early 13th century). The manuscript dates from 1297.



Could you give us a significant example of the encounter between arithmetic and algebra?

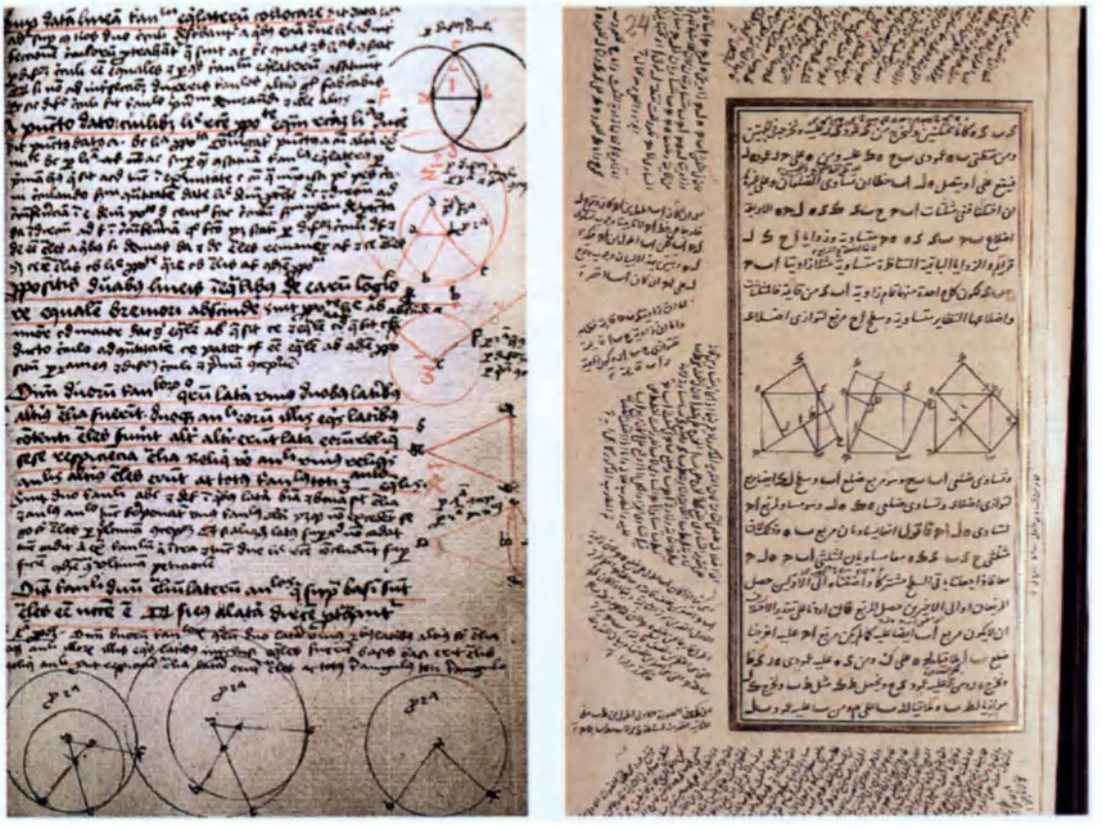
One example that springs to mind is the contribution which Arab mathematics made to classical number theory.

By the end of the ninth century, the most important Greek texts on arithmetic, such as Euclid's own volumes on the subject, the *Introductio arithmeticae* of Nicomachus of Gerasa, and the *Arithmetica* of Diophantus of Alexandria, had all been translated. Further chapters in the theory of numbers were opened in the wake of these translations and could in a sense be said to have been a reply to them. For example, two significant steps were taken in respect of the theory of amicable numbers. The first of these led, in the context of Euclidian arithmetic, to a new set of findings, while, as a result of the application of algebra to the theory of numbers, the second step culminated some centuries later in the creation of a field of number theory that owed nothing to the Greeks. We can look into these two aspects more closely.

Although Euclid put forward a theory of perfect numbers at the end of Volume IX of his *Elements,* neither he nor Nicomachus of Gerasa had developed the theory of amicable numbers.[3] Thabit ibn Qurrah, who translated the work of Nicomachus and revised a translation of the *Elements,* decided to work on this theory. He came up with and demonstrated, in pure Euclidian style, a remarkable formula for amicable numbers which now bears his name.

If we disregard the mystique which had surrounded amicable numbers and look only at the mathematics, it has to be acknowledged that, up to the end of the seventeenth century at least, the history of amicable numbers merely consisted of a passing reference to Ibn Qurrah's formula and its transmission by later mathematicians. These included such Arabic-speaking mathematicians as al-Antaki, al-Bagdadi, ibn al-Banna, al-Umawi and al-Kashi, whose differing origins in both time and place clearly show the widespread dissemination of Ibn Qurrah's formula, which crops up again in the work of Pierre de Fermat in 1636 and in that of René Descartes in 1638.

The second step is noteworthy for the fact that the celebrated physicist and mathematician Kamal al-Din al-Farisi, who died in 1320, wrote a paper in which he deliberately set out to demonstrate Ibn Qurrah's formula, but by a different path. Al-Farisi based his new demonstration on a systematic knowledge of the divisors of a whole number and of the operations that can be applied to them. However, this demonstration involved a reorganization which gave rise not only to a change in the perspective of Euclidian arithmetic, but to the promotion of new topics in number theory. It accordingly became possible to speak of a non-Hellenistic area in number theory.



Right, part of a Latin translation of al-Khwarizmi's treatise on algebra in a manuscript dating from 1145. Far right, a commentary on Euclid's Elements of Geometry by Nasir al-Din al-Tusi (1201-1274). (15th-century Persian manuscript)

In order to make it possible to engage in this new study of divisors, al-Farisi had to establish explicitly certain facts that had only been latent in Euclid's *Elements*. He also had to make use of the advances in algebra since al-Karaji's time in the tenth and eleventh centuries, and especially of combinatory methods. Hence, al-Farisi's approach was by no means confined to demonstrating Ibn Qurrah's formula, but enabled him to embark on a new study involving the first two arithmetical functions: the sum of the divisors of a whole number and the number of those divisors.

This style, which applied algebra and combinatory analysis to Euclidian arithmetic, was still prevalent in Europe in the seventeenth century, at least until 1640. The analysis of al-Farisi's conclusions and of the methods he used thus goes to show that, by as early as the thirteenth century, it had been possible to produce a set of propositions, findings and techniques that had hitherto been ascribed to the mathematicians of the seventeenth century.

What about the new relationship that was established between algebra and geometry?

We have already seen that the mathematical landscape was no longer the same from the ninth century onwards; it was transformed and its frontiers were rolled back. Greek arithmetic and geometry became increasingly widespread. In addition, non-Hellenistic areas were developed within the corpus of Hellenistic mathematics itself. The relationship between the old disciplines was no longer the same, and a host of other groupings were formed. This changing pattern is crucial to an understanding of the history of mathematics generally, for the new relationship between algebra and geometry gave rise to techniques of enormous potential.

The mathematicians of the tenth century embarked on a two-way exercise in conversion which had never previously been envisaged—the translation of geometrical problems into the language of algebra, and vice versa. They translated into algebraic terms solid problems that could not be constructed with ruler and compass, such as the trisection of the angle, the two means, and the regular heptagon. Moreover, algebraists and also geometers such as Abu al-Jud b. al-Leith, when faced with the difficulty of using radicals to solve a cubic equation, were able to turn to the language of geometry and apply the intersecting curves technique to the study of this type of equation.

The first attempts to provide a basis for these conversions were made by al-Khayyam (c. 1048-1131). In his bid to go beyond the specific cases represented by a particular form of cubic equation, al-Khayyam developed a theory of algebraic equations of a degree less than or equal to three which at the same time provided a new model for the formulation of equations. He then studied cubic equations using conic sections in order to reach positive real solutions for them. In order to construct his theory, al-Khayyam had to visualize the new relationship between algebra and geometry in clearer terms before being able to formulate it. From then onwards, the theory of equations appeared, albeit still tentatively, to bridge the gap between algebra and geometry.

In his celebrated treatise *Algebra*, al-Khayyam produced two remarkable findings which historians have wrongly attributed to Descartes. These

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زیکسو رصد خانه مختصر
نمودند نزدیکی آن مقر
درو پانزده اهل علم گزین
شدند از پی خدمت تقی دین
بشد پنج تا فاضل نکته دان

Left, the Istanbul observatory is shown in this Turkish miniature dating from 1581.

Right, title of a manuscript of 1228 which sets forth the theory of amicable numbers formulated by Thabit ibn Qurrah (d. 901).

are the generalized solution of all third-degree equations by means of two intersecting curves, and the possibility of performing a geometric calculation by defining a unit length, which was a fundamental concept.

Some fifty years after al-Khayyam, his successor Sharaf al-Din al-Tusi made a further step forward. In an attempt to demonstrate the existence of the point of intersection of two curves, he arrived at the problems of finding and separating the roots of the equation and of dealing with the conditions under which they existed. In order to find a solution he defined the notion of maximum values for an algebraic expression and tried to find concepts and methods for determining such "maxima".

This not only led al-Tusi to the development of concepts and methods, such as derivatives, that were only to be named as such at a later date, but also compelled him to change his approach. He discovered the need to use local procedures, whereas his predecessors had only considered the overall properties of the objects being studied. All these findings and the theory embracing them are obviously important and have often been attributed to mathematicians who only came several centuries later.

These are the main features of the dialectic between algebra and geometry. To complete the picture, however, two impediments which slowed down the progress of the new mathematics should be mentioned. These were a reluctance to use negative numbers as such, at a time when they had not yet been defined, and the shortcomings of symbolic notation. Both issues were to preoccupy later mathematicians.

In political historiography distinctions are made between Antiquity, the Middle Ages, the Renaissance and modern times. Do you think that this breakdown is relevant in an account of the history of mathematics, and especially of the Arab contribution?

It is true that "medieval" mathematics has been contrasted with "modern" mathematics. The first historical corpus, covering Latin, Byzantine and Arab mathematics, and Indian and Chinese mathematics as well, could be distinguished from another body of work which came into being with the Renaissance. I don't feel that this dichotomy is relevant in either historical or epistemological terms. Arab mathematics clearly represents a continuation and outgrowth of Hellenistic mathematics, which was its breeding-ground. This is also true of the mathematics which grew up in the Latin world from the twelfth century onwards. Finally, the work accomplished in both Arabic and Latin (or Italian) between the ninth century and the early seventeenth century cannot be split into separate periods.

On the contrary, all the indications are that the type of mathematics involved was the same. This is borne out by the fact that we can now compare the work on algebra and numerical computation produced by al-Samaw'al in the twelfth century with that of Simon Stevin in the sixteenth century; al-Farisi's findings in number theory with those of Descartes; the methods used by al-Tusi for the numerical resolution of equations with that of François Viète in the sixteenth century or al-Tusi's search for *maxima* with that of Fermat; al-Khazin's work on integral Diophantine analysis in the tenth century with that of Bachet de Meziriac in the seventeenth century, and so on. If we were to disregard the work

of al-Khwarizmi, Abu Kamil, al-Karaji and others, how could we understand the work of Leonardo of Pisa and the other Italian mathematicians in the twelfth and thirteenth centuries, or the mathematics of the seventeenth century?

True, the later seventeenth century was marked by the emergence of new methods and new areas of mathematical interest in Europe. However, that break did not necessarily occur all of a sudden, nor did it take place at the same time in every discipline. Moreover, the dividing lines seldom coincide with the works of different authors, but often cut right across them. In number theory, for example, the innovation did not lie in Descartes' and Fermat's use of algebraic methods, as has been claimed, since they simply rediscovered al-Farisi's findings. The break can actually be identified within Fermat's own work around 1640, with his invention of the "infinite descent" method and his study of certain quadratic forms.

It was really from the mid-seventeenth century onwards that the tangled threads come together and the main breaks in continuity are identifiable. The contribution made by the Arab mathematicians thus fitted into a coherent pattern which grew up between the ninth century and the first half of the seventeenth century. ■

1. The last five books of Diophantus' *Arithmetica* are principally devoted to the solution of indeterminate equations, i.e. those with more than one variable and a large number of solutions. *Editor*
2. The word *algebra* is derived from the title of this work which was translated into Latin many times in the Middle Ages and had a strong influence on medieval Western science. The word *algorithm*, which designates any method of computation (such as the decimal system) involving a series of steps, is derived from the Latinized form of al-Khwarizmi's name. *Editor*
3. A number is *perfect* if it is equal to the sum of its own divisors (for example 6 = 3 + 2 + 1; 28 = 14 + 7 + 4 + 2 + 1). Two integers form a pair of amicable numbers if the sum of the proper divisors of one is equal to the other. This is true of the numbers 220 and 284, which were for a long time the only known pair of amicable numbers.

The roots of modern maths

BY CATHERINE GOLDSTEIN AND JEREMY GRAY

THE mathematics teaching offered by the European universities around the time when they were founded in the late twelfth and early thirteenth centuries, was largely inherited from the traditions of classical Antiquity. At Paris, Oxford and Bologna, the curriculum covered the quadrivium of arithmetic, geometry, music and astronomy. However as the study of divinity, leading to the elucidation of the basic Christian texts, was considered to be of more importance, the arts faculties, which ran mathematics courses, enjoyed much less prestige than the faculties of law, medicine and above all theology.

The content of maths courses was rudimentary, consisting of the elementary principles of arithmetic and the first books of Euclid's treatise on geometry, the *Elements*. Principles of motion were also studied in the works of Aristotle, and at Oxford and Paris, mathematical methods were beginning to be applied to the study of natural phenomena. The fourteenth-century French bishop and economist Nicole Oresme, for example, attempted to make a graphical representation of variations in speed in a given period of time in certain types of movement.

Measuring the land and charting the heavens

In the late Middle Ages, mathematics was being used by two distinct groups of people, in some cases graduates of the same universities: on the one hand there were tradesmen such as surveyors, engineers and accountants, and on the other there were physicians and astrologers, who had a higher social status.

Economic change in the fourteenth century, the development of the towns and of trade, favoured the rise of mathematicians in society. The increasing complexity of trade called for specialized calculating and accounting skills. Mathematicians gave consultations in "abacus shops", where they drafted contracts and calculations in layman's terms, and resolved banking problems concerning interest rates, the exchange, circulation and value of currency, and the distribution of profits.

In order to make a decent living, these tradesmen had to prove their worth against their competitors and convince their employers or clients that their skills and know-how were useful. Their mastery of the algebraic techniques borrowed from the Arab world was one reason for their professional success.

At first, algebra was not formulated as it is today, by using symbols to express relationships between numbers. It served instead to classify the

L'Académie des sciences et des beaux-arts, an engraving by Gottfried Stein (1687-1747) after a painting by Sébastien Le Clerc.

possible relationships between unknown values and to describe general methods of determining them. These methods were usually set out in treatises as practical examples. The adoption of a system of decimal notation in Europe was both part of and a great incentive to the advancement of mathematics. Texts packed with approximations of roots and complex fractions proliferated, their publication facilitated by the advent of printing.

Thus the growth of mathematics as a skilled profession, whose prestige was linked to that of its chief clients, the merchants, was a corollary of changing economic and social priorities.

New perspectives

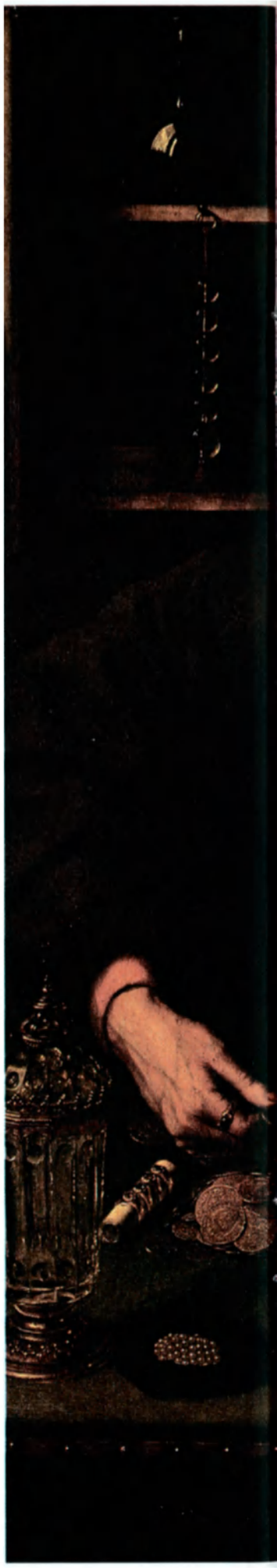
Not only the financial world was in a state of upheaval. A number of practical inventions, such as the compass and binoculars, appeared in the West towards the end of the Middle Ages, and descriptions of measuring instruments and how they worked were sometimes published with texts on algebra. The voyages of discovery, in fact all commercial sea traffic, popularized the use of the astrolabe and the quadrant. Part practical, part theoretical treatises on perspective were written by artists, cartographers and architects, associated with what is now known as geometrical optics. There was no hard and fast division between mathematical theorists and practitioners.

The late fifteenth century saw the rise of the Humanist movement, which concentrated on the study and dissemination of the texts of classical Antiquity. At first the movement did not really favour mathematical methods. If Humanist scholars in the late fifteenth century showed any interest in arithmetic, it was because of their study of Plato or of Pythagoras, but on the whole they despised arithmetical calculation.

Intellectual life, of which the Reformation was one of the most spectacular manifestations, was more concerned with the study of texts than contact with craftsmen. Humanist scholars produced a new batch of translations of key works of Antiquity, from Arabic, then directly from Greek. In going back to the Greek geometers, they were responsible for a revival of interest in mathematics.

The mathematical heritage of Antiquity flourished because it did not fall on barren soil. European rulers increasingly sought the services of scientists, such as the astronomers Tycho Brahe and Johannes Kepler who were attached to the court of Rudolf II in Prague, not only to cast their horoscopes but to construct fortifications or resolve ballistic problems as interest grew in the application of mathematical skills to warfare. Mathematics began to find a place in the largely Humanist education of the aristocracy, and some military engineers were even ennobled as a reward for their services. The growing interest in mathematics in the sixteenth century is typified in the works of such men as John Dee, the English alchemist and astrologer who wrote the preface to the first English translation of Euclid; Gerolamo Cardano, the Italian physician and astrologer whose book on the "Great Art" is one of the cornerstones of the history of algebra; or François Viète, the French jurist who specialized in cryptography and introduced the first systematic algebraic notation.

The indifference, bordering on hostility, which was felt towards applied mathematics began to weaken with the backing of certain reformers of the education system, such as the French philosopher Pierre de la Ramée (Petrus Ramus) who dedicated his teaching chair at the Collège Royal in Paris to mathematics and its practical applications. One of his disciples, the Dutch astronomer and mathematician Willebrord



The Banker and his Wife
by Quentin Metsys (1455-1530).

Snell, was active in the introduction of university mathematics courses in Holland, aided by the foundation of a military engineering school at Leiden. Mathematics, geography and hydraulics competed with and then gradually replaced astrology in European universities.

The rise of the academies

The rediscovery of the works of Greek mathematicians was at its most intense in the early seventeenth century. While translations and reissues were still widely available, many attempts to reconstruct lost or corrupt texts show how widespread mathematical culture was becoming. Mathematics gained in prestige through such events as Galileo's discovery of the satellites of Jupiter, which he named Sidera Medicea in honour of the Medicis, whose family emblem was Jupiter. Galileo believed that mathematics was the key to understanding the language of the universe: "The Book of Nature is written in mathematical characters... the triangles, the circles and the other geometric figures without which it is not humanly possible to understand a single word," he wrote.

Scientific academies modelled on earlier Italian societies also began to appear in Europe. Some of them, such as the Academia dei Lincei to which Galileo belonged, were founded by patrons of the arts and sciences; others were autonomous groups of scholars who met to exchange new books, scientific and literary news, to watch a dissection or to observe the stars.

Printed books and journals, especially scientific ones, were still few and far between, so these academies made a considerable contribution to the dissemination of knowledge. One of the best-known groups met regularly with the Abbé Marin Mersenne in Paris and attracted such eminent scientists as Blaise Pascal, René Descartes, Pierre de Fermat, Gilles Personne de Roberval and Gerard Desargues.

Brought up on the great classical authors, these mathematicians were close to the Humanist tradition. But they were also the heirs of the tradesmen-mathematicians, as shown by the importance attached in their work to algebraic techniques. Some of them were the sons of merchants who had grown wealthy and bought patents of nobility.

Seventeenth-century mathematicians no longer depended on patrons, nor did they live on their earnings from writing or even from teaching. They combined their calling with that of parliamentary adviser, monk, diplomat or soldier. Formerly, masters of the art of mathematics could boost their income by defending their reputation in public. To best an adversary on a mathematical problem would bring in clients. Although mathematicians still offered their interlocutors tricky problems to which only they knew the answers, this seemed to be more of a demonstration of skill or a way of encouraging others to study the subject. Following the lead of Descartes, these men drew a clear line between "geometry", based on reasoning alone, and "mechanics", applicable to the practical demands of craftsmen.

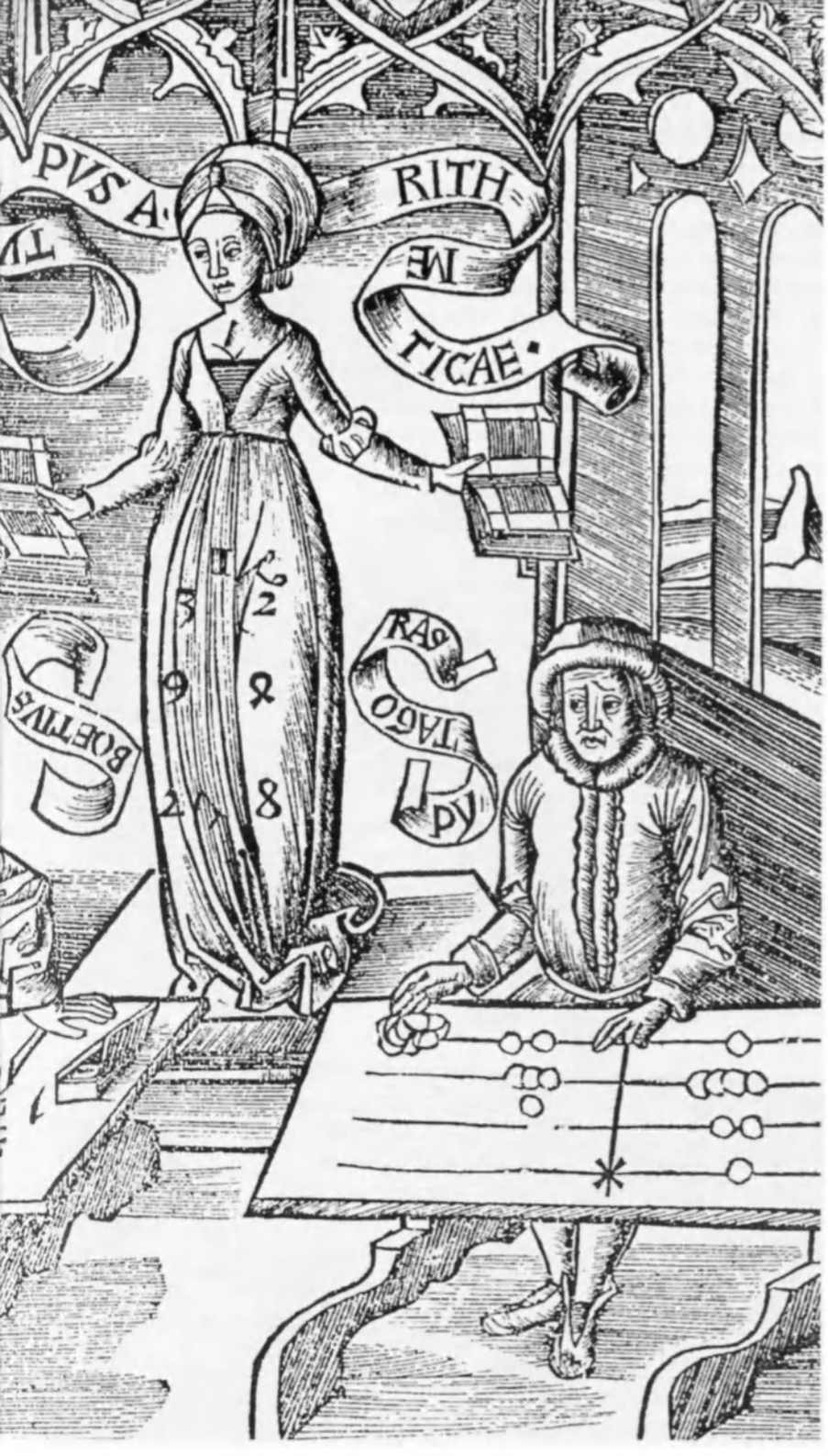
Their geometry did not necessarily resemble that practised by the Greeks, however. Descartes and Fermat turned it into an analytical science. The correspondence and treatises of these men are packed with references to investigations of tangents and curves, calculations of surface area, estimations of probability in games of chance, problems of whole numbers. They were also interested in optics, in fact in all natural phenomena—now held to be governed by mathe-

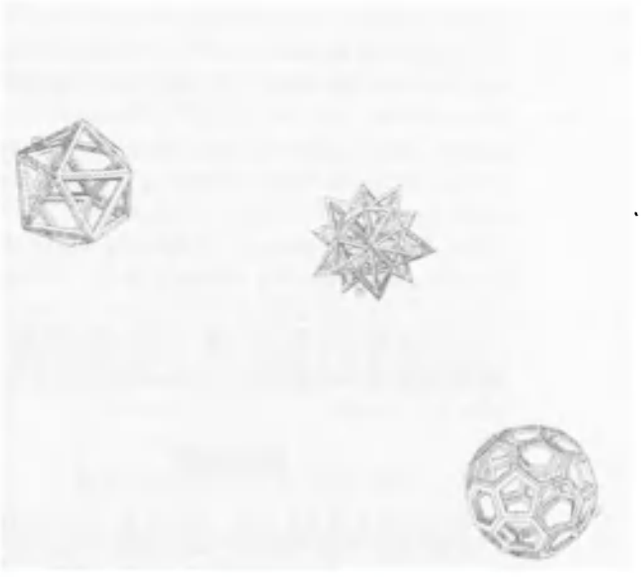
In this wood engraving of 1503, arithmetic (centre) seems to be settling the dispute between "abacists", the supporters of calculation using counters, and "algorists" who used written notation, in favour of the latter.

The Pythagorean *quadrivium*

The writings of Plato (and of Isocrates) tell us of the generally accepted organization of the mathematical sciences that was current during the Hellenistic era. In the medieval educational system it was called the *Pythagorean quadrivium*. It consisted of arithmetic, geometry, astronomy and music. Astronomy could mean either elementary notions about the rising and setting of stars, the calendar and the seasons, or more elaborate mathematical notions concerning the real and apparent movements of the Sun, the Moon and the planets. The strictly mathematical aspects were sometimes called Spherics, owing to hypotheses concerning the spherical shape of the Earth and the cosmos that were current from the fifth century BC. Similarly, music in this context signified the theory of musical intervals, in particular the numerical determination of consonant intervals. It was also known as canonics. It seems that this system existed from the fifth century BC—perhaps indeed linked to the Pythagorean school—in the context first of "higher", then of "secondary" education.

Bernard Vitrac





"Isaac Newton, very great head of school but not pompous" is the title of this Japanese engraving attributed to Hôshû (c. 1890).

matical laws. This scientific rigour, a trend which began at Oxford in the fourteenth century, had mechanics as its model.

In the mid-seventeenth century, the state took over from private initiative to support, at least in part, the development of the sciences. In 1662 a number of academies in England were incorporated by royal charter into the Royal Society of London, and the French Académie des Sciences, founded by the minister Jean-Baptiste Colbert in 1666, counted among its early members those who had worked with Mersenne. Periodicals such as the French *Journal des Savants* and the Royal Society's *Philosophical Transactions* made the latest scientific discoveries known to all who were interested.

The second half of the seventeenth century saw an increase in studies based on analytical geometry. Other mathematicians' discoveries relating to methods of calculating area and tangents were synthesized, independently, by the English physicist Isaac Newton and the German philosopher Gottfried Wilhelm Leibniz, each of whom elaborated a different but in each case coherent theory of differential and integral calculus. Neither of their theories was wholly acceptable to their contemporaries and a major row went on for years over the priority or the superiority of both contenders.

These developments marked a new phase in the relationship between mathematics and the physical world. In his masterpiece *Philosophiae*

Naturalis Principia Mathematica (The Mathematical Principles of Natural Philosophy) published in 1687, Newton set forth a unified theory of planetary motion expressed in geometrical terms similar to those used in Euclid's *Elements.* This theory, on which numerous commentaries were written in the following century, led to greater understanding of natural phenomena. In the 1730s, following study of Newton's work, the French mathematician Alexis Claude Clairaut specified the periodicity of certain planetary phenomena, including that of the celebrated Halley's Comet (named after the British astronomer who was the first to calculate its orbit).

The Age of Enlightenment

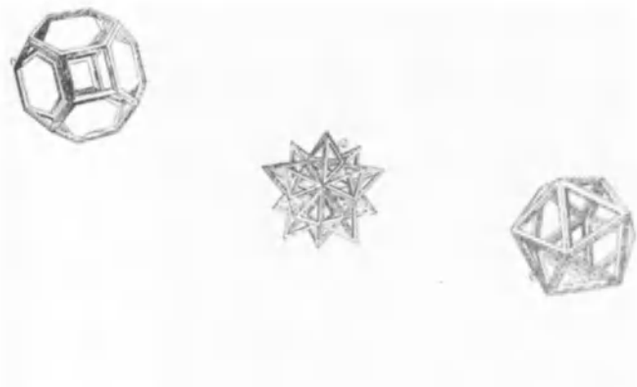
The tradition established at the end of the seventeenth century was carried on in the eighteenth. But while soldiers and diplomats, knights and nobles, continued to show an interest in mathematics, they were gradually overshadowed by professional mathematicians looking for full-time employment. In some countries, mathematicians were attached to educational institutions, in others they belonged to scientific academies or royal courts. Preference was given to the practical applications of mathematics and their contribution to the advancement of technology and to human welfare. In France, especially, the Enlightenment encouraged the spread of scientific ideas that would ensure the "march of progress" and "free minds from religious prejudice". The French mathematician and encyclopaedist Jean Le Rond d'Alembert wrote a number of articles on mathematics for the *Encyclopédie.* Men too circulated, as well as ideas. When the Swiss physicist Leonhard Euler moved from the Prussian Academy of Sciences at Berlin to the St. Petersburg Academy, the Italian-French mathematician Joseph-Louis Lagrange left Turin to fill Euler's post at the invitation of Frederick II the Great, on whose death Lagrange went to the court of Louis XVI in France.

Even more systematically than in the previous century, mathematicians were writing, exchanging theses and problems, and publishing articles on research in progress as well as finished treatises. They broadened the scope of the calculus and sought to increase its applications. Euler's *Introduction to Infinitesimal Analysis* (1748), was one of a number of didactic works written by leading mathematicians to make the new ways of thinking accessible to students. They suggested methods of notation and standardized the problems considered to be of interest. Confidence was growing in algebra as a tool. Calculus no longer sought justification in geometry and at last became an independent discipline.

The rediscovery, assimilation and surpassing of the Graeco-Arab heritage thus spanned several generations of social change and intellectual liberation. It came about through men whose interests ranged from new techniques of drainage or construction, to cosmogony and methods of calculating area. ■

CATHERINE GOLDSTEIN, French mathematician, is a researcher with the National Centre for Scientific Research (CNRS). Her research is mainly concerned with the theory of elliptical curves and the history of the theory of number in the 17th century. She was a contributor to *Eléments d'histoire des sciences* (Bordas publishers, Paris, 1989), prepared under the direction of Michel Serres.

JEREMY GRAY, of the United Kingdom, teaches mathematics at Britain's Open University. He has published several works on the history of modern maths, including *Ideas of Space* (Oxford University Press, 2nd ed., 1989) and recently co-edited (with John Fauvel) an anthology entitled *The History of Mathematics and the Reader* (MacMillan, London, 1987).



The solar system according to Galileo is shown in this anonymous 16th-century engraving.

GABRIELA MISTRAL

POET AND HUMANIST

THE life of Gabriela Mistral, who was born on 7 April 1889 in a village in northern Chile and died in New York in 1957, was devoted to an intellectual and spiritual quest. From her early days in Chile's Elqui valley to her European travels on cultural and diplomatic assignments, the story of her career reads almost like a myth. The needy peasant girl becomes the doyenne of Latin-American literature. The humble rural schoolteacher is awarded some of the world's highest honours, including the Nobel Prize for Literature in 1945.

Gabriela Mistral's poetry, from her 1922 collection *Desolación* ("Desolation") to *Lagar* ("The Wine Press") of 1954, was written in harsh, powerful and colloquial language. Like her massive output of prose, it is informed by a visionary, prophetic sense of the destiny of Latin America. But readers in Europe and countries as culturally diverse as Israel, China and Japan, also found a meaning in the humanism and poetry of her work.

In many books, theses, poetic and philosophical reflections, it is possible to trace the influence of this Latin-American writer from a country which, within a mere half century, produced three writers of world stature: Gabriela Mistral, Vicente Huidobro and Pablo Neruda.

Why has Gabriela Mistral's work had a universal impact? As in the case of all true creative artists, attachment to her own familiar world did not exclude a strong feeling for other languages and cultures. She acknowledged her debt not only to Saint Theresa and to the Spanish poet Luis de Góngora y Argote but to Dante, Rabindranath Tagore, and the great Russian writers, and, Christian though she was, to the great sages of Buddhism.

Closely identified with her country and with her people ("I am and will remain," she said, "a daughter of my land"), Gabriela Mistral described her personal experience with a voice which all humanity could recognize, drawing from a tragic love affair a song of love and tenderness which speaks to people everywhere. In her sympathy for the downtrodden and her readiness to defend their cause, poetry and humanism become one: "We must give expression to the soul in all its intensity, and boldly utter the message which springs from the heart before it ceases to beat." ■

The above text has been adapted from an article by Mr. Gastón Von Dem Bussche, professor at the Metropolitan University of Chile.

An Academy of Sciences for the Third World

BY AKHTAR MAHMUD FARUQUI

SCIENCE has been described as the highest expression of internationalism, the common preserve of the "haves" and the "have-nots". Yet its diffusion has been dramatically uneven. While science in the industrially developed nations of the North goes from strength to strength, in the South scientists are relatively few. They work in isolation from the world scientific community and the resources at their disposal are scanty.

In the last few years a new institution, the Third World Academy of Sciences (TWAS), has emerged as a leading contributor to efforts to bridge the science gap between North and South. The Academy, which is based in Trieste, Italy, is the first international forum to unite men and women of science from the Third World with the aim of promoting basic and applied sciences in developing countries.

The Academy is the brainchild of Nobel Laureate Abdus Salam of Pakistan, who has devoted more than forty years of his life to helping scientists from impoverished developing countries overcome their isolation and participate in advanced scientific work at the frontiers of knowledge.

The headquarters of TWAS are in Trieste, Italy, on the premises of the International Centre for Theoretical Physics (ICTP), which is celebrating its twenty-fifth annniversary this year and which Professor Salam was also instrumental in founding and has headed since its inception.

First mooted by Professor Salam in 1981, the Academy was officially launched in July 1985 by the Secretary General of the United Nations, Mr. Javier Perez de Cuellar.

In the four years since it was founded, the Academy has floated and implemented a variety of schemes to bolster South science. It has awarded some 300 research grants to scientists working in a broad spectrum of fields including optics, nuclear physics, solar energy, seismology and laser physics. Applicants, whose projects are vetted by four international referees, have been drawn from many countries—Jamaica, China, Argentina, India, Brazil, Pakistan, Mexico, Angola, Peru, Thailand, Turkey and Madagascar, to name just a few.

Through a donation programme run in collaboration with ICTP, the Academy also provides scientific literature to developing countries facing shortages of foreign currency. A joint appeal to libraries, publishing companies, laboratories and individuals to donate books, journals and equipment to developing countries has met with an encouraging response. Under the programme, some 50,000 books and journals are being supplied annually to some 500 institutions in 90 countries.

One important TWAS project is a South-South Fellowship Programme designed to enable a scientist from one developing country to work in another. So far 125 of these Fellowships have been awarded. The Academy also supports visits by Third World scientists to biology, chemistry and geology laboratories in Italy.

In 1987 the Academy instituted a History of Science Prize to highlight the work of a scientist from a Third World country whose achievements had not been previously recognized. The first prize was awarded in 1988 for an essay on the astronomical tables which were compiled by the great mathematician Shams al-Din al-Khalili and used in Damascus for timekeeping from the fourteenth century to the nineteenth. The competition, which is open to scholars of any nationality, carries a cash prize of $10,000.

TWAS has also awarded annual prizes of $10,000 to outstanding scientists from Third World countries in the fields of physics, chemistry, mathematics and biology, and organized conferences on major problems confronting the Third World. A Conference on the Role of Women in the Development of Science and Technology in the Third World, held in October 1988 in Trieste, was attended by 240 women scientists from over 70 countries.

A non-governmental, non-profit-making organization, TWAS is currently funded largely by Canada and Italy. Its members, ten of whom are Nobel Laureates of Third World origin, consist of 106 Fellows from 42 developing countries, 42 Associate Fellows (scientists from industrialized countries who were either born in developing countries or have distinguished themselves in Third World science) and 3 Corresponding Fellows.

In late 1988, only three years after its creation, Ronald Léger of the Canadian International Development Agency commented that TWAS had already brought significant changes to many Third World scientists working in isolation. "For them," Léger pointed out, "TWAS has meant spare parts for a lab, or travel grants and South-South exchanges for young scientists, or a subscription to a scientific journal for an isolated scientist. Above all, TWAS has meant a new hope for science and technology in developing countries." ■

AKHTAR MAHMUD FARUQUI is editor of the Newsletter of the Third World Academy of Sciences.

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42nd YEAR

Published monthly in 34 languages and in braille by Unesco, The United Nations Educational, Scientific and Cultural Organization.

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IMPRIMÉ EN FRANCE
(Printed in France) - DEPOT LEGAL : C1 - NOVEMBRE 1989.
Photocomposition: The *Unesco Courier*
Photogravure-impression: Maury Imprimeur S.A., Z.I. route d'Etampes, 45330 Malesherbes.

ISSN 0041-5278 — N° 11 - 1989 - OPI - 89 - 1 - 474 A

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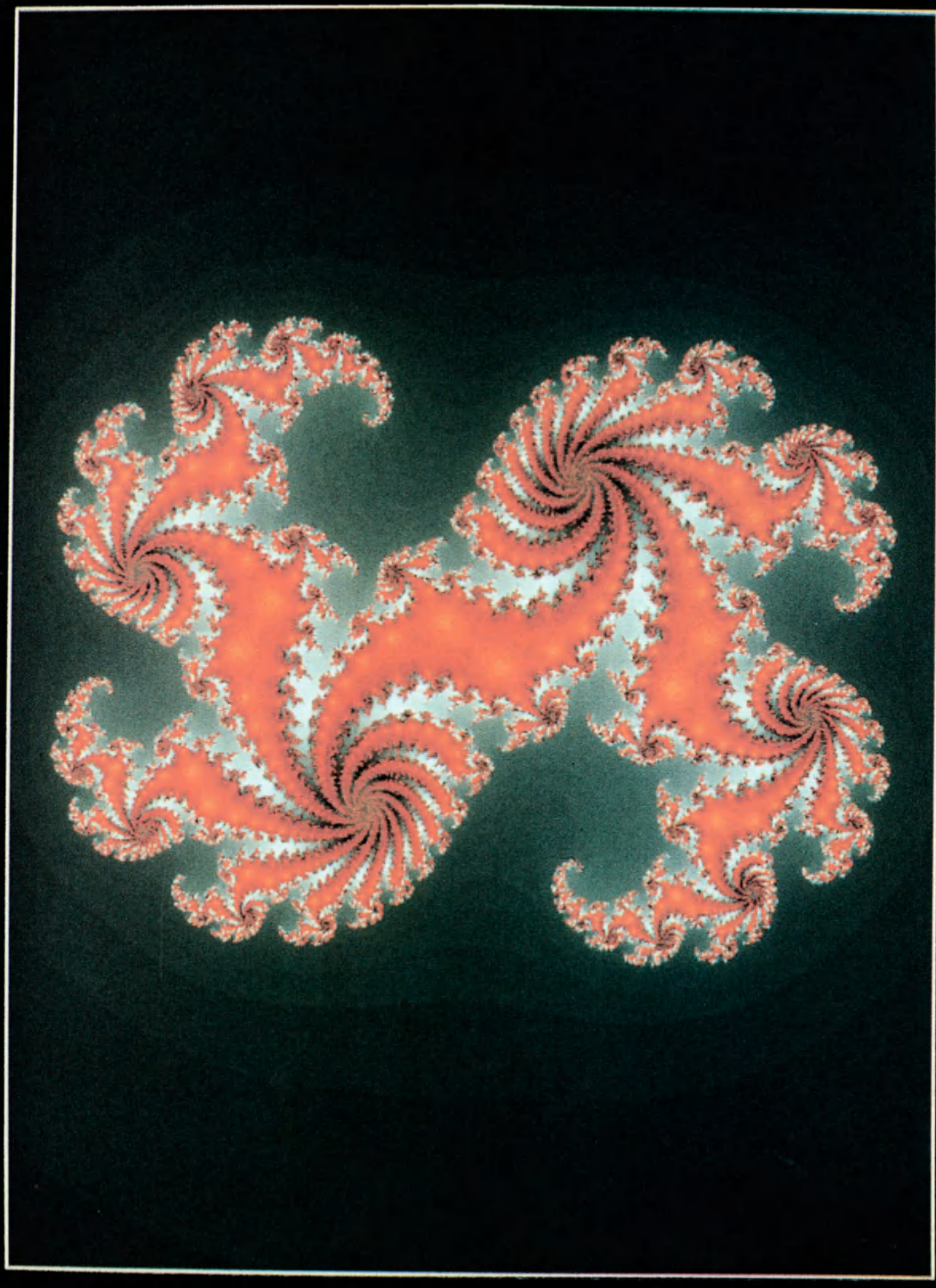
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